Meteorology 432

Precision, Standards, Error & Error Propagation

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Precision

• Reliability or repeatability of a measurement.
  – How close together or how repeatable results are.

• Checked by repeating measurements.
  – Typically reported by using standard deviation.

• A precise instrument will give nearly the same reading every time.

• Poor precision results from poor technique.
Accuracy

• Correctness.
  – How close a measurement is to the expected value.

• Checked by using different methods.

• Poor accuracy results from procedural or equipment flaws.
Accuracy vs. Precision

• “Accuracy is telling the truth…precision is telling the same story over and over again.” – Yiding Wang
Accuracy vs. Precision

Precision vs. Accuracy
Types of Error

• Systematic
  – Cause an instrument to be off by a similar magnitude.
  – Defective apparatus, incomplete working equations, human observers, etc.

• Random
  – Introduces errors that can result in slightly different measurements.
  – Present in all measurements.
Random Error

• Present in all measurements

• Arise from sudden or uncontrollable changes within the measured environment.

• Random act of carelessness by observer.

• Errors usually likely to be positive or negative.

• Minimized by taking a large number of measurements.
Sources of Error

- Static Errors
- Dynamic Errors
- Drift Errors
- Exposure Errors
Static Error

• Input is held steady or slowly varying.

• Errors remain after calibration.

• Deterministic or Random
  – Deterministic: Hysteresis, residual non-linearities, sensitivity to unwanted input.

Dynamic Error

• Errors due to changing input
  – Usually rapidly varying input.

• Disappear when input is constant long enough for output to stabilize.

• Usually assessed after static errors have been determined.
Drift Errors

- Physical changes that occur in a sensor over time.
- Frequent calibration.
Exposure

• Due to imperfect coupling between sensor and the atmosphere.

• Examples: Thermometer
  – Thermometer will never be exactly at the air temperature.
  – Thermometer interacts with its environment.
Thermometer Exposure Errors

• Radiation

• Conduction

• Static Conditions
Limiting Exposure
More Exposure

• Cannot be accounted for during calibration.

• Statements about instrument accuracy and precision do not include exposure error.

• Exposure error can easily exceed all other error sources.
  – For a properly calibrated, well designed data acquisition system that is well maintained, exposure is the largest source of error.

• Instruments report their own state.
  – To some extent, any platform interacts with its environment.
Standards

- Calibration
- Performance
- Exposure
- Procedural
Calibration

• Standards maintained by NIST.
  – National Institute of Standards and Technology.

• Maintain standards for temperature, humidity, pressure, wind speed, etc.
Performance

• Standard method of testing sensors to determine their performance.

• Consists of definition of terms and methods of testing static and dynamic performance.
Exposure

• Where should instruments be placed?
  – Anemometers mounted on sides of buildings?
  – Roof?
  – Height of instruments?

• WMO Standards
  – World Meteorological Organization.
Surface Wind Standard

• Height: 10 meters

• Distance between anemometer and obstruction must be 10 times the height of the obstruction.
Surface Temperature Standard

• Height: 1.25 meters and 2.00 meters above ground surface.

• Radiation shield
  – With or without forced ventilation.
  – Setup must not be shielded by buildings and/or trees.

• Sensor must not be placed on a steep slope or in a depression.
Procedural

• Selection of data sampling and averaging periods.
Why Standards?

• Reliability of measurements
  – Are you really measuring what you think you are measuring?

• Measurement comparisons

• Any other reasons?
Significant Figures

• The number of significant figures is the number of digits needed to state the result of a measurement without losing precision.

• Addition and Subtraction: Round result to last decimal place of least precise result.

• \[ 10.01\text{cm} + 352.2\text{cm} + 0.0062\text{cm} = ??\] → \[362.2\text{cm}\]

• \[362.2162\text{cm} \rightarrow 362.2\text{cm}\]
More Sig. Figs.

- Multiplication or division: round to the smaller number of significant figures.

- Example: Rectangle of L=63.52cm, H=3.17cm
  - Find area.

- Area = 63.52cm x 3.17cm = 201.3584 cm$^2$

- Area = 201 cm$^2$

- Note: It is generally advisable to retain all available digits in intermediate calculations and round only the final result.
Representation of Error

• Errors should only be given to one significant figure.

• Relative error = error / measured quantity
  – Fractional error

• Percentage error = (error / measured quantity) * 100
  – Relative error * 100
Distributions

• Take a measurement, $x_1$, of a quantity, say $x$.
  – Do we expect our measurement to be exactly equal to $x$?

• Take another measurement, $x_2$, of $x$.
  – Do we expect this measurement to be exactly equal to $x$?
  – Do we expect this measurement to be equal to $x_1$?

• As we take more and more measurements, we expect a pattern to emerge.
  – If we can eliminate systematic errors, they should be distributed around $x$. 
Distributions

• If we make an infinite number of measurements, we would know the exact distribution.
  – This distribution is a result of random errors.
  – This distribution is called the parent distribution.

• Of course, this is not possible. The measurements we make are sample of the parent distribution.
  – Sample distribution.

• Histogram
  – Can represent raw number of times a value appears, or its frequency (frequency plot).
Histogram
Probability Distribution

- Take each histogram bin and divide by total number of measurements.
- What does a bar mean now?
- It is the probability that you will obtain a measurement in that range.
- If we normalize to the total area under the curve, we get a probability density function, $P(x)$.
- What is $P(x) \, dx$?
- The probability that a randomly selected measurement will yield an observed value of $x$ within the range of $(x - dx/2) \leq x < (x + dx/2)$.
- What is the area under the curve?
Mean, Median, Mode

- **Mean**

- **Median**: value for which half the observations are less than median, and half are greater than the median.
  - Cuts the probability density function in half.

- **Mode**: Most probable value
  - Value for which the probability distribution has its greatest value.
  - Value most likely to be observed.

- *We can say something special about these three for a symmetrical distribution, what is it?*
Deviations

• $d_i$ of any measurement, $x_i$, from the mean $\mu$ is defined as the difference between $x_i$ and $\mu$.

• If $\mu$ is the true value of the quantity, $d_i$ is the error in $x_i$.

• Standard deviation: measure of the dispersion, or spread of the observations.
  – Variance.
Summary

• Consider the **mean** to be the **best estimate** of the true value under the prevailing experimental conditions.

• The variance and the standard deviation characterize the uncertainties associated with the experimental attempts to determine the true value.
Gaussian Distribution

- Gaussian, or normal, distribution describes the distribution of random observations.
- Typically described in terms of the mean and the standard deviation.
Propagation of Errors

• Often want to determine a dependent variable that is a function of one or more measured variables.

• How do the errors or uncertainties in the measured variables carry over or propagate to determine the uncertainty in the dependent variable?

• In practice, we estimate the error or uncertainty in a variable through its standard deviation.
  – How do we combine standard deviations?
Error Propagation Equation

• First two terms: reflect how the uncertainties in u and v contribute to the uncertainty in x.
  – These two terms dominate.

• Third term: average of the cross terms involving products of the deviations in u and v.
  – If u and v are uncorrelated, we should expect to find equal distributions of positive and negative values under the summation sign.
  – On average, these will cancel out eliminating this term.
More Significant Digits

• Rule for Stating Uncertainties: Experimental uncertainties should almost always be rounded to one significant figure.

• Rule for Stating Answers: The last significant figure in any stated answer should usually be of the same order of magnitude (in the same decimal position) as the uncertainty.
Example

- A student wants to measure the acceleration of gravity, \( g \), by measuring the time \( t \) for a stone to fall from a height, \( h \), above the ground. After taking several timings, she concludes that:

\[
    t = 1.6 \pm 0.1 \text{ s}, \quad \text{and} \quad h = 46.2 \pm 0.3 \text{ ft}
\]

- What does she get for \( g \)?