An Analysis of the Rossby Wave Theory

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ABSTRACT

Large-scale planetary waves are known as Rossby waves. The Rossby wave theory gives us an idealized model for the movement of these waves, which can ultimately determine large-scale weather patterns. Using 500 mb observations, this theory was tested and analyzed. Results indicate that Rossby theory does a decent job of predicting wave movement using the wave number and upper level wind speeds, but it would be beneficial to incorporate wave amplitude, since it also appears to have an affect on the wave propagation.

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1. Introduction

Rossby waves are planetary waves largely driven by vorticity advection. These waves are largely responsible for air mass movement and, ultimately, local weather phenomena. Two parts of Rossby waves affect the local weather of a region: first is the wave amplitude, which is responsible for the strength of the system; and second is the wave speed, which is responsible for the movement of the system (Holton, 2004).

The next obvious question we would have is how we can go about solving for the amplitude and wave speed using data that is easily attainable. Using the laws of basic atmospheric dynamics and the fact that Rossby waves are driven by vorticity, an equation can be derived for the speed of these waves. We then find that the wave speed is ultimately related to the number of waves, the coriolis parameter, and the wind speed:

$$c - u = -(\beta)/K^2$$

The amplitude of the wave is not as quantitatively analyzed as speed is for Rossby wave theory. We know that the digging and lifting of the troughs and ridges is due to the vorticity advection and the speed of the upper-level winds. Since vorticity is difficult to measure, finding a relationship between the wave amplitude and the speed of the upper-level winds is the best method.

The final step in analyzing Rossby waves is to figure out how Rossby theory relates to real life. What does the theory us about the evolution of large-scale waves? Do our observations agree with this theory? What are some ways to improve upon the model based on real-life observations? These are the questions we will be exploring in more detail throughout the report.

2. Data and Methodology

In the weather community, weather maps are created and analyzed daily. For this project, 500mb heights and zonal winds images were acquired through the Iowa State Weather Products page (http://www.meteor.iastate.edu/wx/data/). These images were used to analyze patterns and record data of the wave and wind patterns in both the Northern and Southern Hemispheres from 22 August 2005 to 11 November 2005. Five variables were collected/calculated for each hemisphere: wave number, amplitude, wave speed, maximum zonal wind speed at 500mb, and maximum zonal wind speed between 150 and 300mb. For the first three variables a target contour was used as a reference to analyze the waves. In the Northern Hemisphere the 5580m height contour was used and in the Southern Hemisphere the 5280 meter height contour.

Using 50N or 50S latitude on the 500mb height images, the wave number (N) was calculated by the times the target contour crossed that latitude and divided by two. The average amplitude (A) was determined by calculating the mean maximum height of the waves on a particular day and the mean minimum heights. The average amplitude is then calculated by averaging the mean maximum and minimum. The average amplitude is then calculated by averaging the averaged maximum and minimum. The speed of the waves (C) is measured by following a wave from the day before a particular day to the day after. The difference in the longitude is calculated and the speed is determined by dividing it by 2. A better estimate would be to average the speed of all current waves, however our method was used to avoid outlier speed values.

The zonal wind maps were used to determine the maximum winds at specified levels for both the northern and southern hemispheres. At 500mb the maximum wind and corresponding latitude was determined for both the northern and southern hemisphere. The maximum was also determined in the 150 to 300mb level, with approximate latitudes, in both hemispheres .

3. Wave Comparison Summary

a. Northern Hemisphere

There are four periods where the wave number remains the same for four or more days. The period from early-mid September to mid October is a period where the wave number oscillates between 2 and 4 waves. Typical synoptic time scale is approximately 1-3 days, which aligns similarly to what is found in Figure 1 The amplitude of the waves does not appear to be a function of wave number in the Northern Hemisphere according to Figure 2. The wave amplitude increases slightly throughout the time series (Figure 3). The amplitude increased by about 33% from the end of August to November.

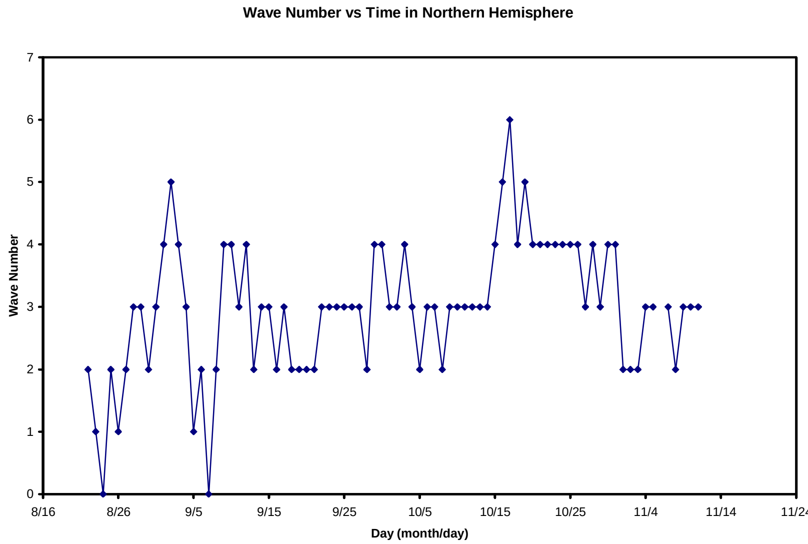

FIG. 1.The pattern of wave number for each day of the data shows the variability in the waves at 500mb in the Northern Hemisphere.

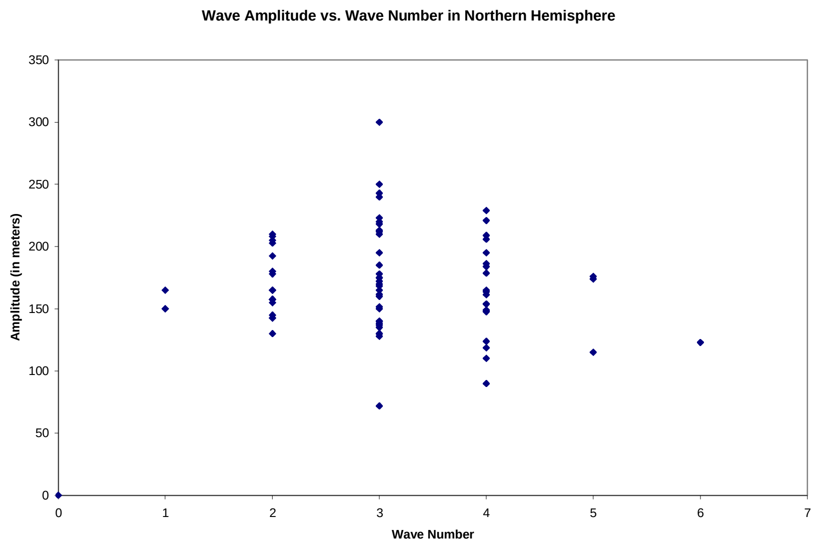

FIG. 2.Wave amplitude versus wave number shows little relationship between the two variables.

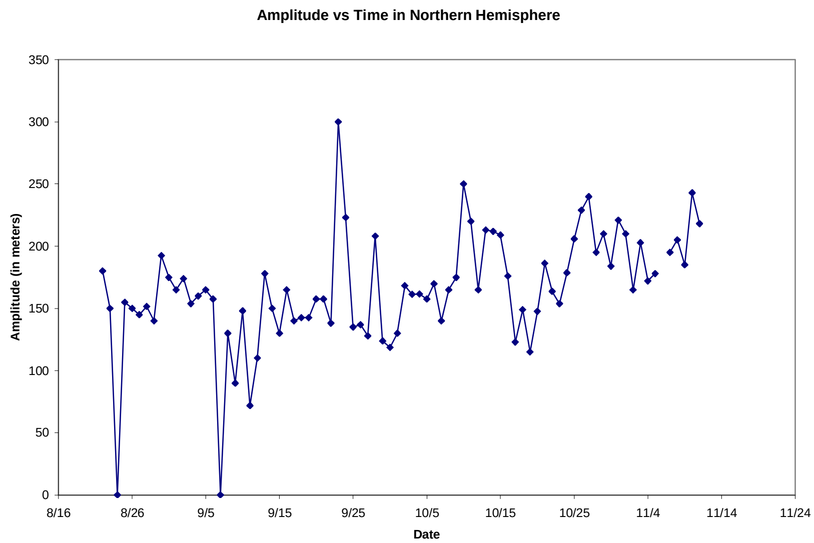

FIG. 3.The wave amplitude as a function of time shows a slight increase in time of amplitude. The two data points of 0 are missing data points.

b) Southern Hemisphere

In Figure 4, there are three periods that maintain the same wave number for four or more days. From mid September through early October the wave number varies by +/- 3. Figure 5 shows a slight relationship between wave number and amplitude. It appears a smaller wave number results in a larger amplitude. This could be due to a space constraints that keeps higher wave number's amplitudes from becoming to large. There appears to be no general trend of the wave amplitude over the time series. Figure 6 shows the random pattern of the wave amplitudes.

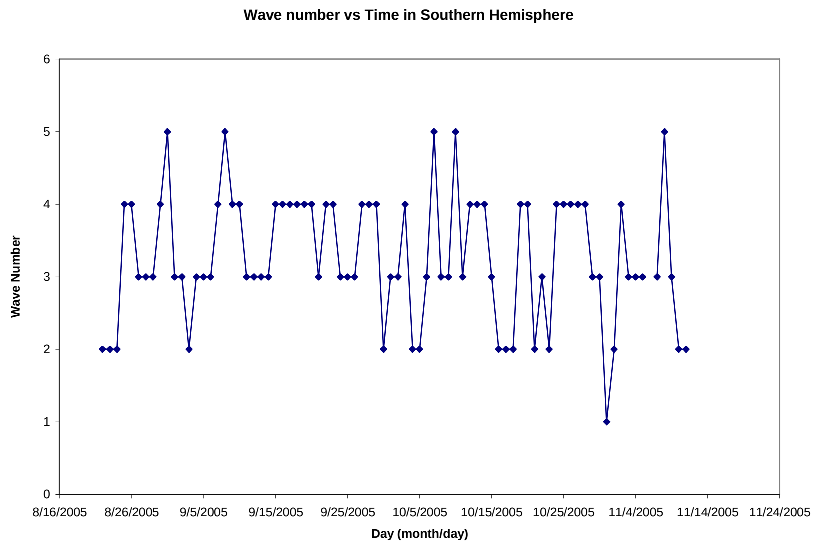

FIG. 4.The pattern of wave number for each day of the data shows the variability in the waves at 500mb in the Northern Hemisphere.

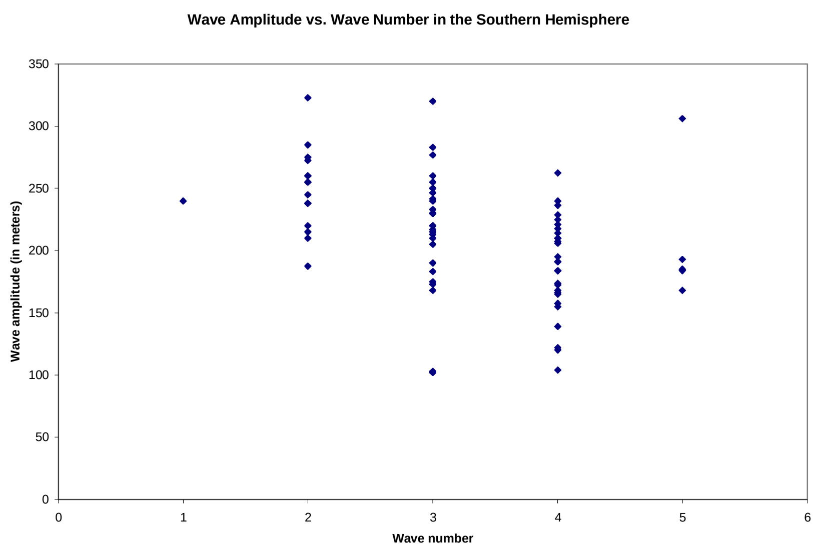

FIG. 5.Wave amplitude versus wave number in the southern hemisphere. A slight relationship exists with smaller wave numbers resulting in higher ampltitudes.

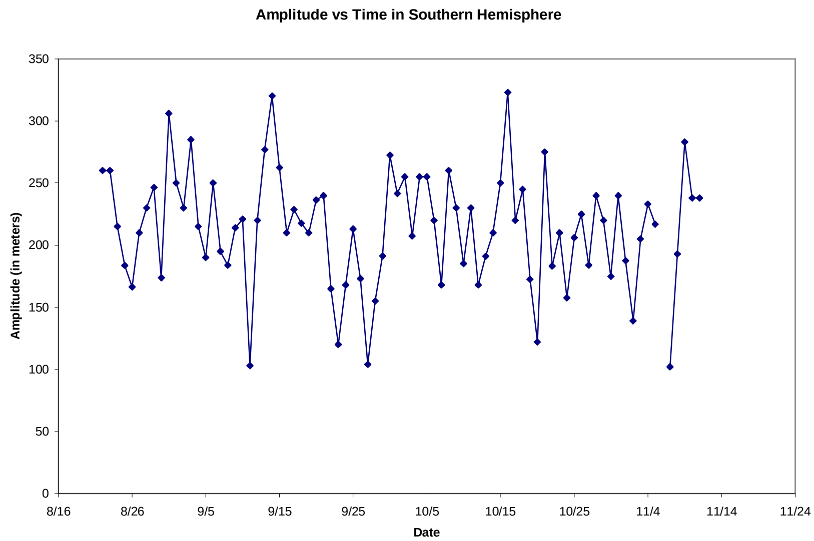

FIG. 6. There appears to be no relationship between wave amplitude and time.

4. Zonal Wind Evolution and Wave Amplitude

Understanding wind evolutions is important due to how it can affect a forecast, which can belong range (such as 7 days out) or climatological (such as 2-3 months out). Shifts in zonal wind speeds and patterns can affect temperatures over a certain latitude; which can, in turn, affect the weather specific to an area at that latitude.

As expected, each zonal wind evolution graph turned out different, depending level and hemisphere. As shown in figure 6, the time period was approximately three months from late August to mid November.

a. Northern Hemisphere: Upper Wind Plots

Upper level zonal winds in the Northern Hemisphere fluctuated between 30 m/s and 50 m/s continuously for the 3-month period. The one exception to this was in early October when most values were at either 20 m/s or 60 m/s and one at 65 m/s. (Figure 7)

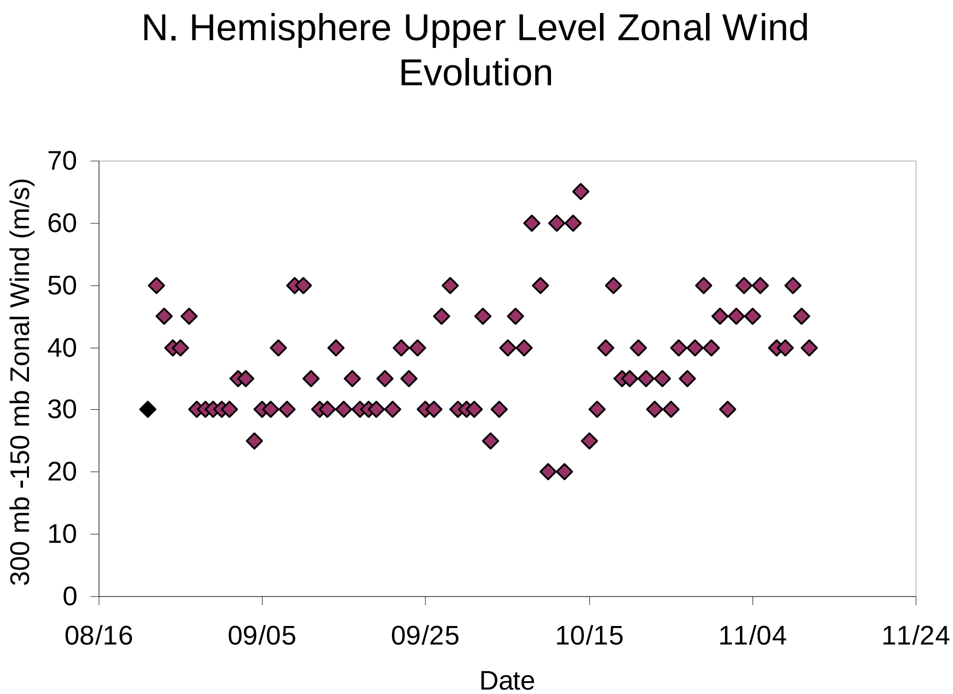

FIG. 7. Evolution of Upper Level Zonal Wind Speeds in the Northern Hemisphere

b. Northern Hemisphere: Mid Level Wind Plot

Mid level zonal winds in the Northern Hemisphere were not as consistent as the upper level winds. The minimums were fairly consistent at 15 m/s, but the maximum values were 25 m/s from late August to early September,30 m/s throughout most of September, and 35 m/s in October with two days getting up to 40 and 45 m/s. In November, the winds dropped back down to 25 m/s. (Figure 8)

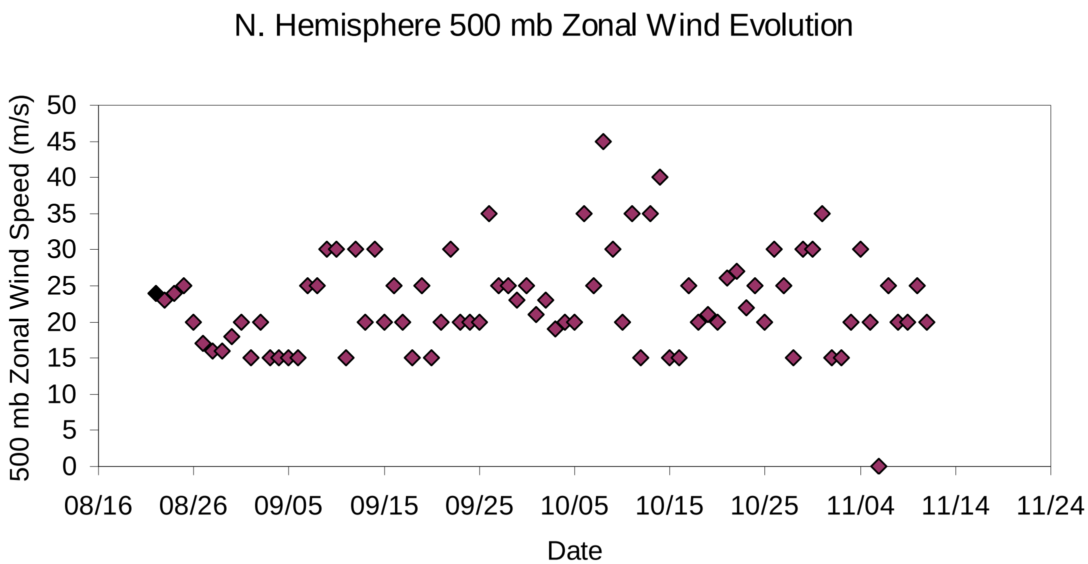

FIG. 8. Evolution of Mid Level Zonal Wind Speeds in the Northern Hemisphere

In general, both levels show consistency in that the winds reached their highest speeds at approximately the same time period in early October.

c. Southern Hemisphere: Upper Level Winds

As time goes on, zonal winds vary from 40 m/s to 75 m/s at first, and then they start to vary from about 40 m/s to 60 m/s. Near the end of the three-month period, values remained fairly consistant at or near 45 m/s. (Figure 9)

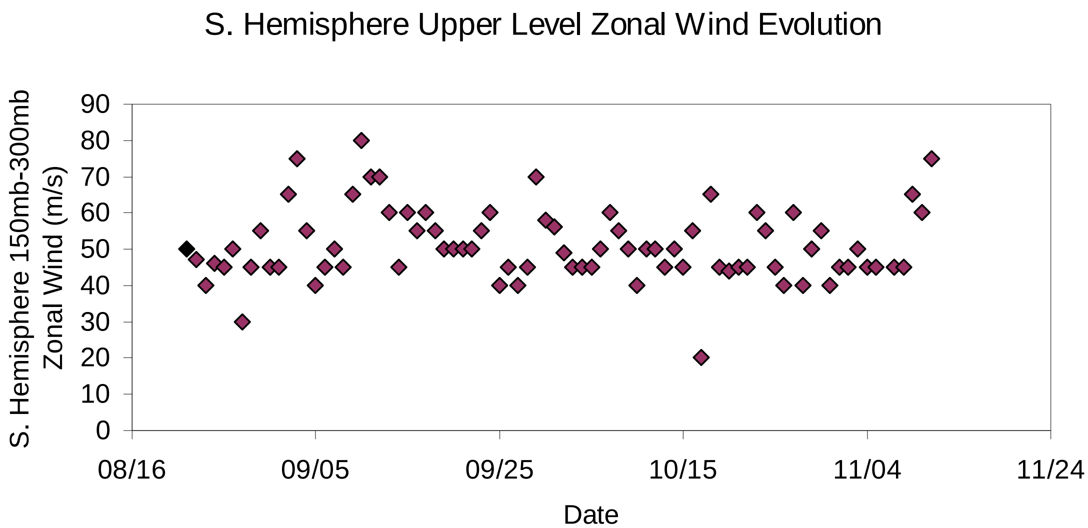

FIG.9. Evolution of Upper Level Zonal Wind Speeds in the Southern Hemisphere

d. Southern Hemisphere: Mid Level Winds

As with the Northern Hemisphere mid level winds, the minimum values stayed about the same, with the maximum values fluctuating over the period. In late August, the highest value was about 37 m/s, but the maximum winds sped up to 45 m/s in September. After this, the winds generally slowed down to about 30 m/s in early October. Yet, local maximum occurred in late October with the highest values at 40 m/s, which is where they generally remained constant for the remainder of the three-month period. (Figure 10)

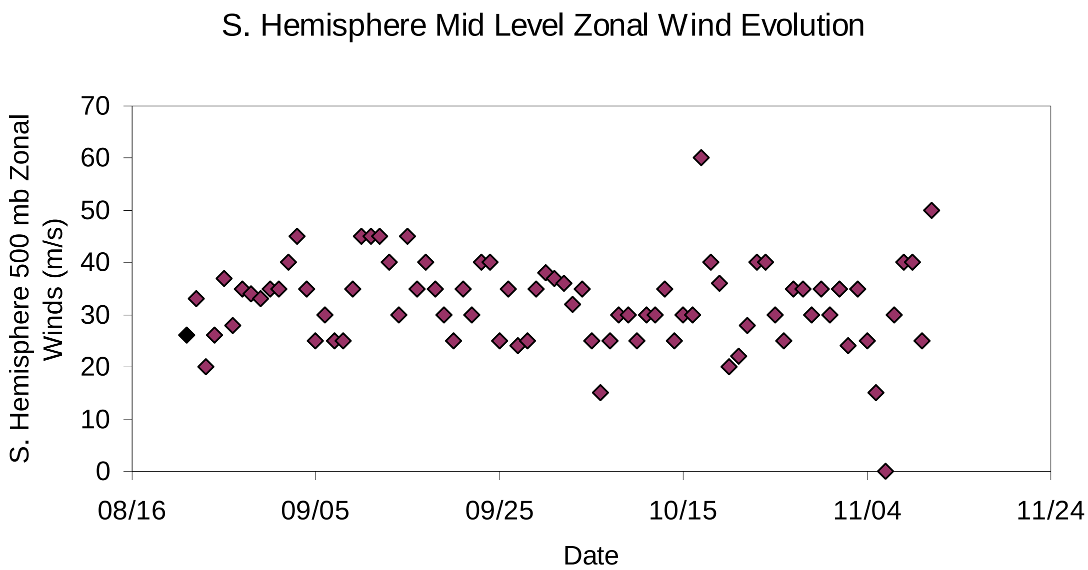

FIG. 10. Evolution of Mid Level Zonal Wind Speeds in the Southern Hemisphere

There were very few noticeable relationships between zonal wind speed and wave growth/decay. However, there appeared to be a relationship between the fluctuations of both. Amplitudes fluctuated the most when zonal wind speed fluctuated between their highest and lowest values. This occurred consistently at both levels and in both hemispheres(Figures11-14).

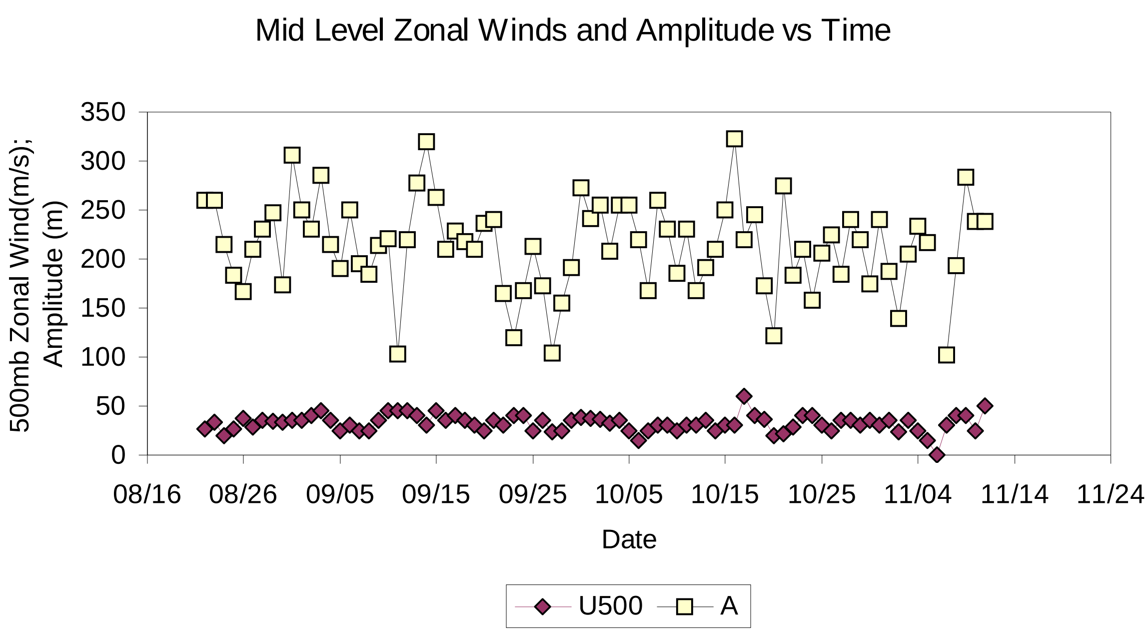

FIG. 11. Mid Level Zonal Wind Speed and Amplitude vs Time in the Southern Hemisphere

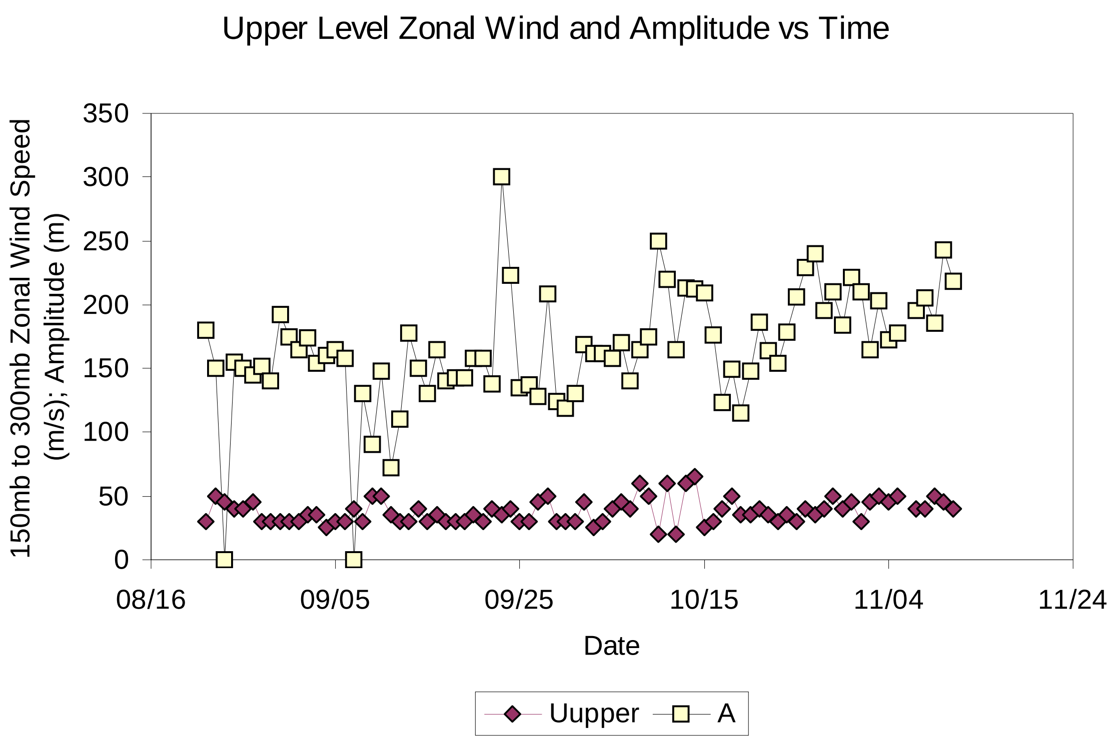

Fig. 12. Upper Level Zonal Wind Speeds and Amplitude vs Time in the Southern Hemisphere

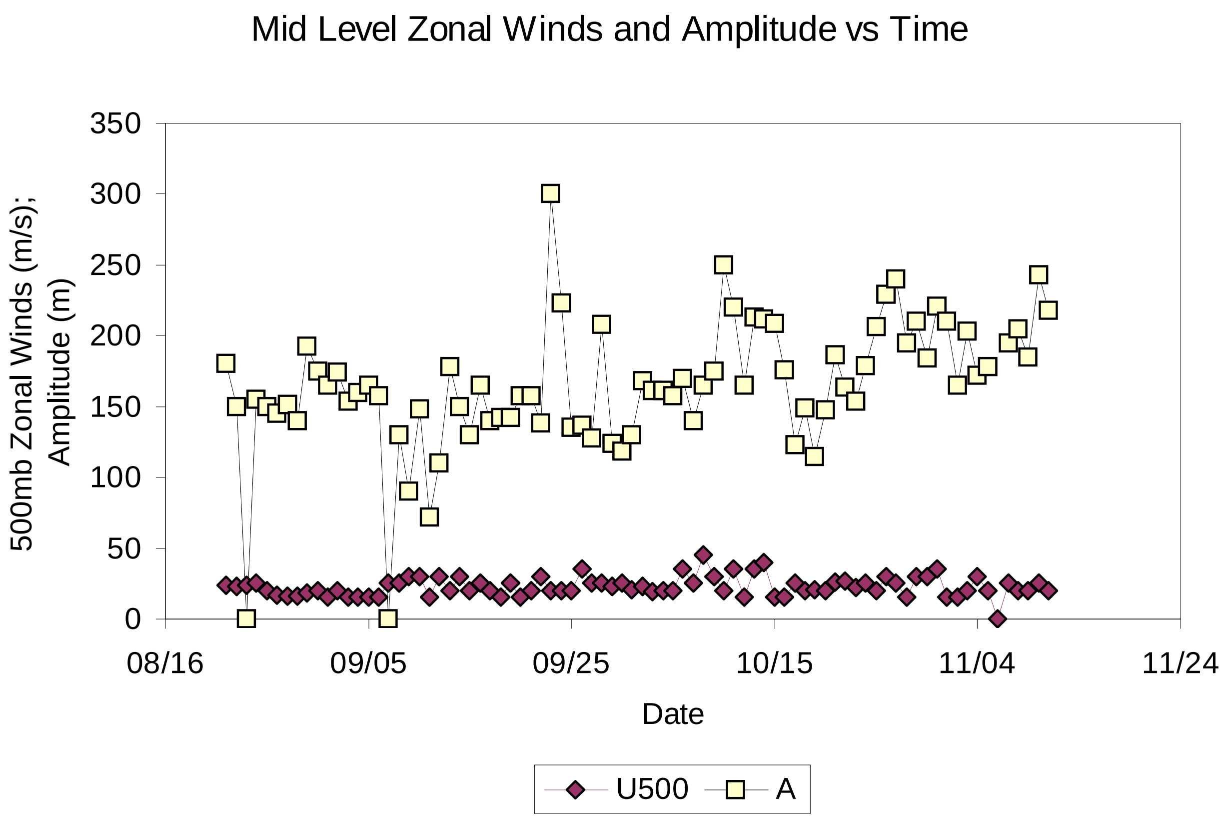

Fig. 13. Mid Level Zonal Wind Speeds and Amplitude vs Time in the Northern Hemisphere

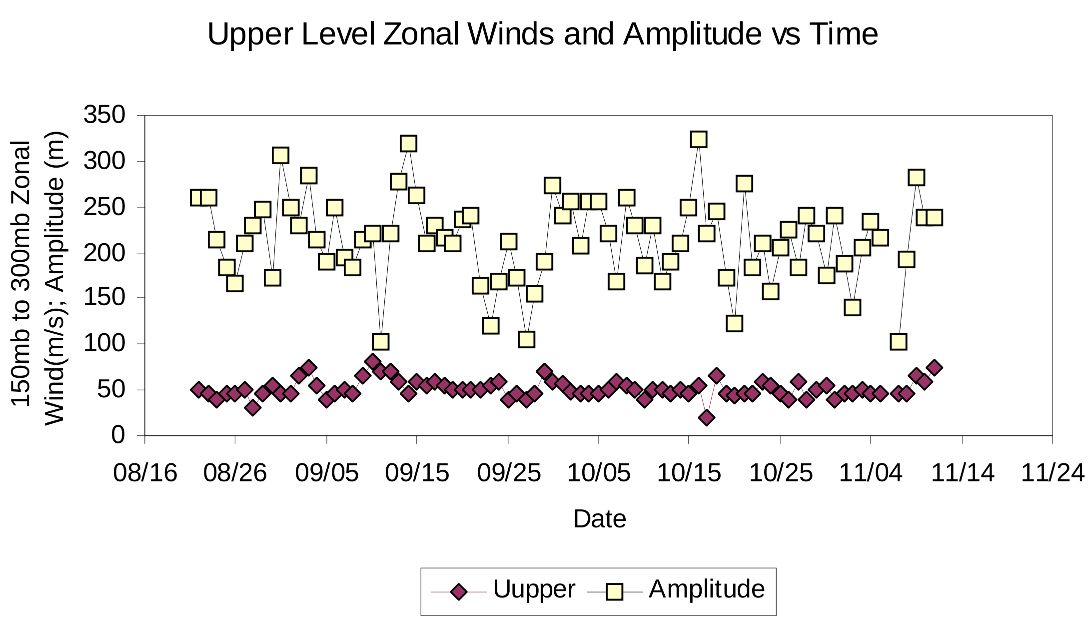

Fig. 14. Upper Level Zonal Wind Speeds and Amplitude vs Time in the Northern Hemisphere

5. Wave Propagation Comparison

To do some of these calculations, we made the assumption that 1degree longitude is approximately equal to 150,000 m at 45 degrees latitude. This was obtained by using the border of Kansas as a reference point. We know Kansas is about 111 km from north to south, so measuring the longitude at 45 degrees latitude we found it to be about 150km.

a. 500 mb Winds

In the Northern Hemisphere, waves propagate from west to east; while in the Southern Hemisphere, it's from east to west. Regardless of direction, both hemispheres' waves generally travel at the same approximate speed, which is20 degrees/day. Using the assumption that 1 degree latitude is approximately 150 km, we can convert the speed to be about 3000 km/day, which would also tell us that one wave would probably take about 18 days to travel 360 degrees around the entire earth (assuming this happens at about 45 degrees latitude).

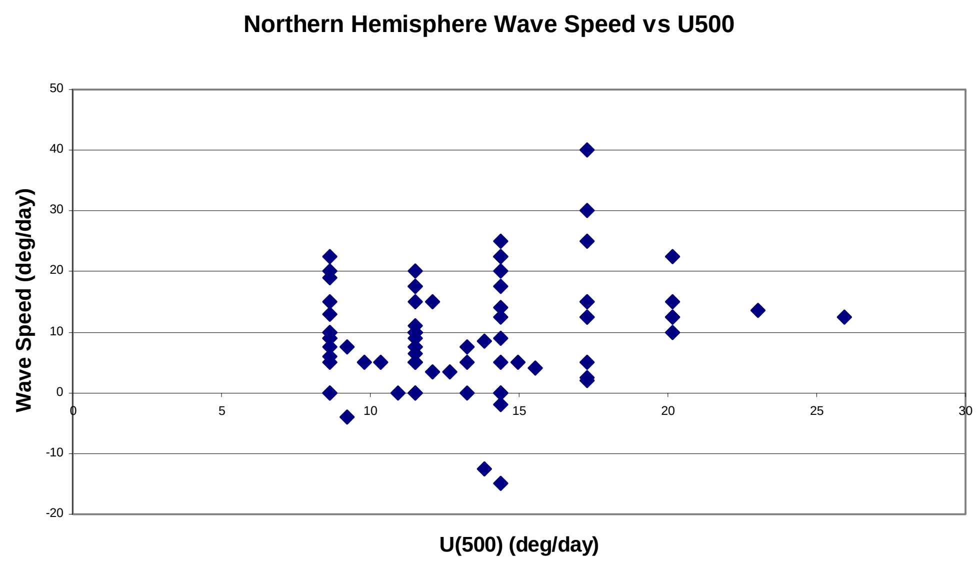

FIG. 8. The comparison of maximum wind speeds at 500 mb with the wave speed in the Northern Hemisphere.

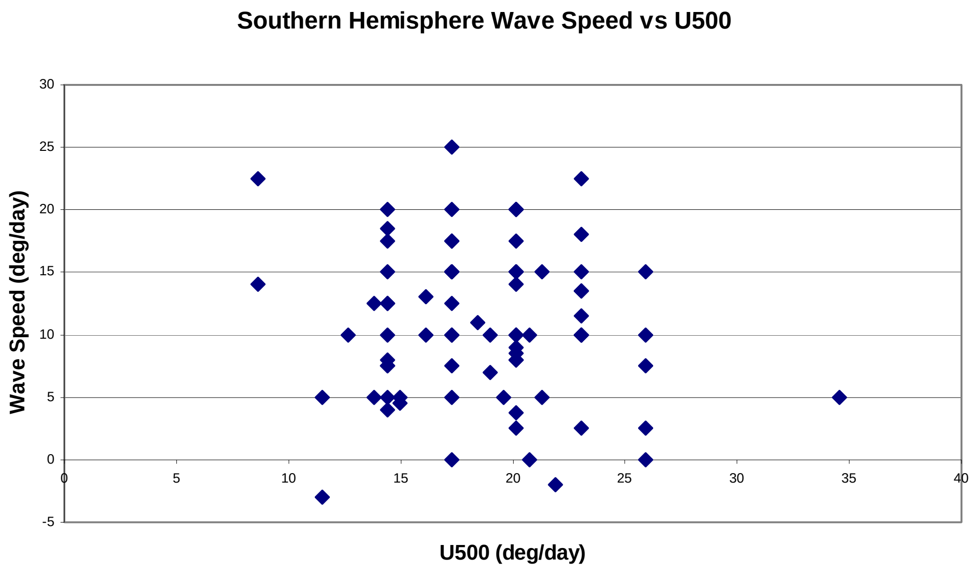

FIG. 9. The comparison of maximum wind speeds at 500 mb with the wave speed in the Southern hemisphere.

Rossby wave theory can be somewhat useful in a situation like the plot in Figure 16. Holton (2004) states that the theory equates the wave speed to the wind speed with the equation

$$c - u = -(\beta)/K^2.$$

Where c is the wave speed, u is the wind speed, β is the variation of the Coriolis force with latitude, and K is the total horizontal wave number. According to this theory, there should be a relationship between the wave speed and the wind speed in some form of nonlinear function. In our graph, the relationship is clearly not linear, but there appears to be a relationship to some extent. Thus, Rossby theory could be a fit to this data, but more exploration is required.

b. Upper Level Winds

Rossby wave theory suggests that the relationship plotted in Figures16 and 17 make a little more sense. Since c is smaller, we end up with a negative number on the left hand side of the equation, which cancels out the right hand side of the equation. This indicates that the wave speed is in the same direction as the wind speed, which is consistent with what we saw in our data collection. The actual relationship is still unclear, however, because it does not allow us to determine what type of regression we have.

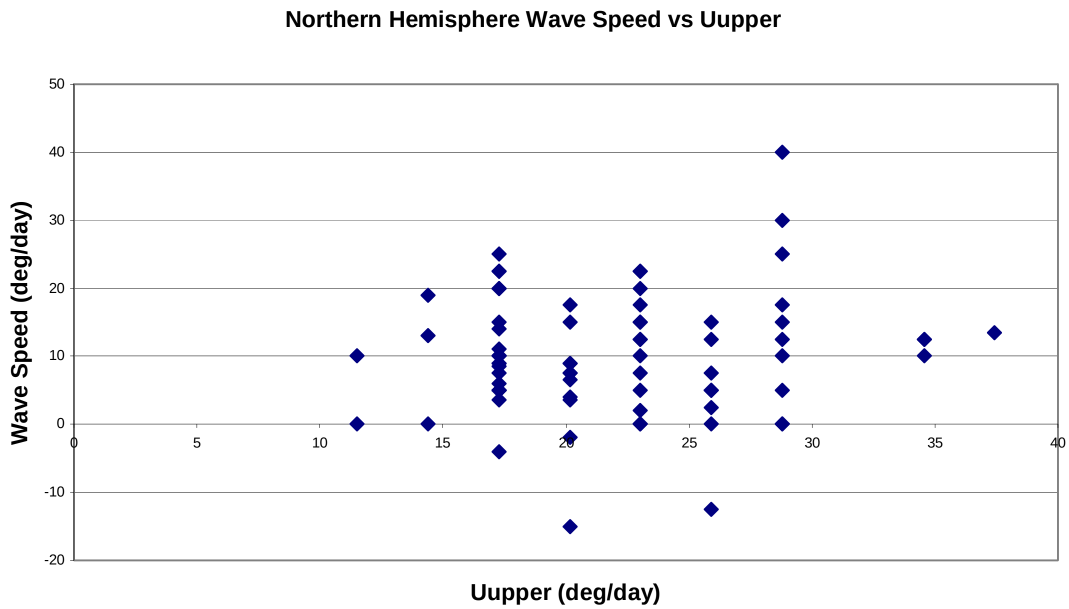

FIG. 10. Comparison between the maximum upper level winds (between 300mb and 150 mb) and the wave speed in the Northern Hemisphere.

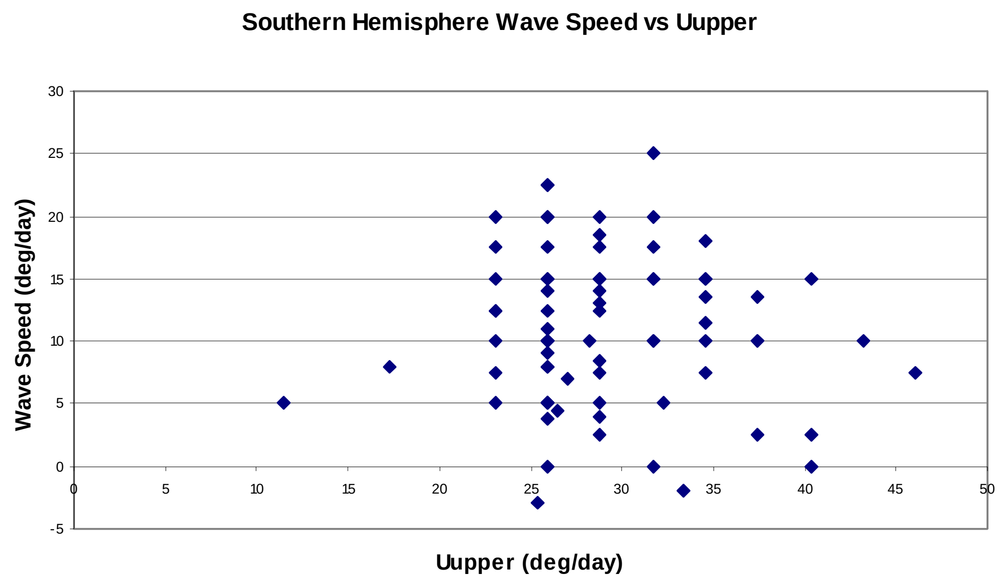

FIG. 11. Comparison between the maximum upper level winds (between 300mb and 150 mb) and the wave speed in the Southern Hemisphere.

It appears that there could be a fairly strong relationship with this plot because the data appears to “stack” on each other, inferring that there are a lot of similarities. It is a slightly more complicated relationship, though, much like the last plot. Thus, we still need more analysis to find the correct relationship to speed.

c. Wave Speed as a Function of 500 mb winds

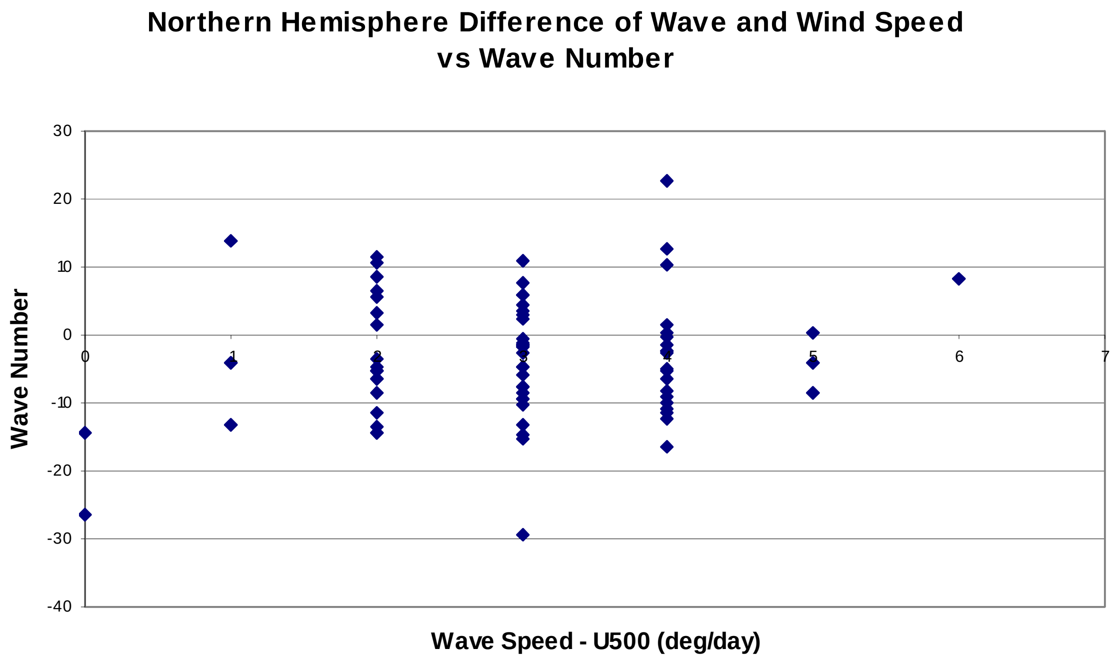

FIG. 12. The function obtained by subtracting the 500 mb winds from the wave speed in comparison to the wave number in the Northern Hemisphere.

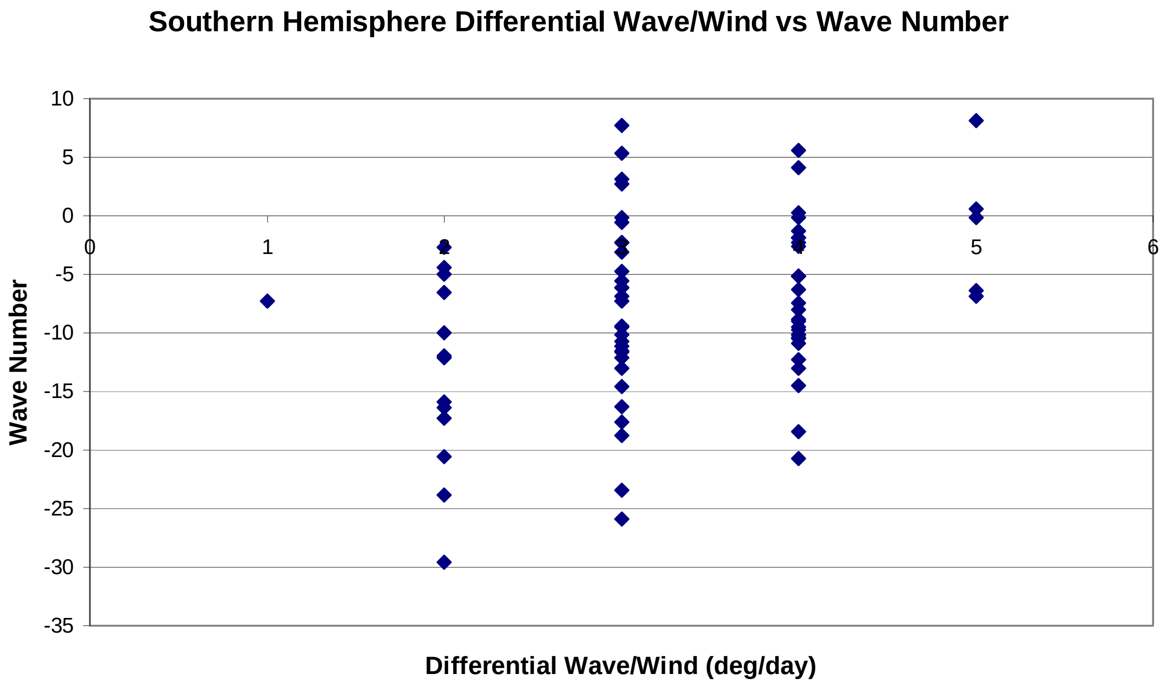

FIG. 13. The function obtained by subtracting the 500 mb winds from the wave speed in comparison to the wave number in the Southern Hemisphere.

This is the strongest relationship of all the plots we tried. It appears to be fairly linear, but with higher Difference values as the wave number increases, indicating a slightly exponential relationship. The interesting difference between the Northern and Southern Hemisphere in this comparison is the orientation on the plot. It appears that the negative values dominate in the Southern Hemisphere, whereas positive values dominate in the Northern Hemisphere. This makes sense because the direction of propagation in the Northern Hemisphere is in the opposite direction of the direction of propagation of the Southern Hemisphere. More importantly, there were periods of time in the Northern Hemisphere when the waves retrograded, i.e. moved backwards. This did not appear to happen at all in the Southern Hemisphere.

Rossby theory more or less makes sense with this comparison because the difference between the wind speed and wave speed is negatively proportional, with the wave number being a squared value. Thus, we can qualify that the relationship we see in the graph is similar to what Rossby theory suggests, though it is a little difficult to tell for sure.

6. Conclusions

Rossby wave theory tells us what speed the waves are moving and can relate it to upper level wind speeds. For the wave speed, the best graphical representation of the Rossby theory occurs when the plot of 500 mb winds minus the wave speed was compared to the wave number. This appears to agree with Rossby theory, however the exact relationship is difficult to pinpoint when comparing a graph like Figure 20 to an equation. It is clear from sections 3 and 4 that to improve upon Rossby wave theory, the amplitude should be incorporated into the equation.

References

Holton, J.R., 2004: *An Introduction to Dynamic Meteorology*. 4th ed. Elsevier Inc., 511 pp.

ISU, cited 2005: ISU Meteorological Products [Available online at http://www.meteor.iastate.edu/wx/data.]