

Problems - 1

Due: 20 Sept. 2018

You are welcome (in fact, encouraged) to discuss problem sets with your classmates. However, each of you must turn in your own problem-set answers.

1. In the shallow-water model, Thompson (1961) arrives at an approximate condition for a wave's phase speed under the further assumption that

$$(\bar{u} - c)^2 \ll g\bar{h}$$

Specifically, he shows that

$$(c - \bar{u}) = - \frac{\beta + f^2 \bar{u} / g \bar{h}}{k^2 + f^2 / g \bar{h}}$$

(Note that Thompson uses α to represent the wavenumber k , but k is more common.)

Recognizing that $k = 2\pi/L$, where L is the wavelength of the wave, compute $(c - \bar{u})$ for several wavelengths between $L = 10^5 \text{ m}$ – $L = 10^7 \text{ m}$ with sufficient resolution to plot $(c - \bar{u})$ as a function of L .

(a) Do this for a midlatitude case, using

$$f = 10^{-4} \text{ s}^{-1} \quad \beta = 1.6 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \quad g = 9.8 \text{ m s}^{-2} \quad \bar{u} = 10 \text{ m s}^{-1} \quad \bar{h} = 10^4 \text{ m}$$

(b) Which wavelengths would have retrograde motion?

(c) Suppose $\bar{u} = 30 \text{ m s}^{-1}$. How does that change the plot, and what wavelengths now have retrograde motion?

(d) Do this for an equatorial case (plot on the same graph), using

$$f = 0 \quad \beta = 2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \quad g = 9.8 \text{ m s}^{-2} \quad \bar{u} = 10 \text{ m s}^{-1} \quad \bar{h} = 10^4 \text{ m}$$

(e) Which waves move more rapidly [have larger magnitude of $(c - \bar{u})$]?

(f) Now consider these waves on Mars, plotting on the same graph. Use

$$f = 10^{-4} \text{ s}^{-1} \quad \beta = 3.0 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \quad g = 3.7 \text{ m s}^{-2} \quad \bar{u} = 10 \text{ m s}^{-1} \quad \bar{h} = 1.5 \times 10^4 \text{ m}$$

(g) How do Martian wave speeds compare with Earth's?