$$\frac{\partial \Phi}{\partial z^*} = R\bar{T}/H$$

$$z^* = -H \ln(p/p_0)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - S_p\,\omega = J/C_p$$

$$S_p = \frac{RT}{C_p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta}\frac{\partial \theta}{\partial p}$$

$$\left(\frac{\partial}{\partial t} + \vec{V}\cdot\nabla\right)\frac{\partial\Phi}{\partial z^*} + w^* N^2 = \kappa J/H$$

$$N^2 = \frac{R}{H}\left(\frac{\partial T}{\partial z^*} + \frac{\kappa T}{H}\right)$$

Multiply by $\frac{R}{H}$:

$$\left(\frac{\partial}{\partial t}+\vec{V}\cdot\nabla\right)\left(\frac{RT}{H}\right)+\frac{1}{\Theta}\frac{\partial\Theta}{\partial p}\left(\frac{R\omega}{H}\right)=\frac{RJ}{c_p H}$$

$$\left(\frac{\partial}{\partial t}+\vec{V}\cdot\nabla\right)\frac{\partial\Phi}{\partial z^*}+\frac{R}{H}\left(\frac{T}{\theta}\frac{\partial\theta}{\partial p}\right)\omega$$

$$=\kappa J/H$$

ω term:

$$\omega^*=\frac{-H\omega}{p}$$

$$\frac{R}{H}\left(\frac{T}{\theta}\frac{\partial\theta}{\partial p}\right)\omega = -\frac{R}{H}\left(\frac{T}{\theta}\frac{\partial\theta}{\partial p}\right)\left(\frac{p}{H}\right)\left(\frac{-H}{p}\right)\omega$$

Note:

$$p \frac{\partial(\ )}{\partial p} = \frac{\partial(\ )}{\partial \ln p}$$

Because

$$d(\ln p) = \frac{1}{p}\, dp$$

$$= -\frac{R}{H}\left(\frac{T}{\theta}\right)\left(\frac{1}{H}\frac{\partial\theta}{\partial \ln p}\right)w^*$$

$$dz^* = -H\, d\ln p$$

$$= + \frac{RT}{H}\left(\frac{\partial \ln\theta}{\partial z^*}\right)\omega^*$$

$$\frac{\partial \ln\theta}{\partial z^*} = \frac{\partial}{\partial z^*}\left\{\ln(p_u^\kappa) - \ln p^\kappa + \ln T\right\}$$

$$\theta = (p_0/p)^\kappa T$$

for the whole

$$\frac{RT}{H}\left\{-\kappa\frac{\partial \ln p}{\partial z^*}+\frac{1}{T}\frac{\partial T}{\partial z^*}\right\}$$

w^*

$$\frac{\partial \ln p}{\partial z^*}=-\frac{1}{H}$$

$$= \underbrace{\frac{R}{H}\left\{\frac{\kappa T}{H} + \frac{\partial T}{\partial z^*}\right\}}_{=N^2} w^*$$

So then

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \nabla\right) \frac{\partial \overline{\Phi}}{\partial z^*}$$

$$+ \qquad N^2 w^* = \frac{\kappa J}{H}$$

2

$T(20\,km) = 200$

$T(50\,km) = 260$

$\nabla = +2\,K/km$

$$z^*(20km) = 20km$$

what it gives

$$z^*(50km) = 50km$$

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{p}{RT} g$$

$$-H \frac{\partial h p}{\partial z} = +\frac{gH}{RT}$$

$$\frac{\partial z^*}{\partial z} = gH/RT$$

$$\int_{z=20\,km}^{z=50\,km} dz^* = \int_{20}^{50} \frac{gH}{RT}\, dz$$

$$z^*\Big|_{20\,km}^{50\,km} = \frac{gH}{R} \int_{20}^{50} \frac{dz}{T(z)}$$

$z^*(50\,km) - z^*(20\,km)$

$= 30\,km$ for LHS

$T(z)$? $\quad T(z) = 160 + \Gamma z$

$\Gamma = 2\,K/km$

“Type a quote here.”

–Johnny Appleseed

RHS

$$= \frac{gH}{R} \int_{20}^{50} \frac{dz}{160 + 2z}$$

$$= \frac{gH}{R} \left(\frac{1}{2}\right) \ln(160 + 2z) \Big|_{20}^{50}$$

$$30\,\text{Km} = \frac{gH}{gR}\ln\left(\frac{260}{200}\right)$$

Γ

0.2624

$$H = \frac{\Gamma R}{g(0.2624)}\,30\,\text{Km} = 6.7\,\text{Km}$$