Downslope Wind Events

Stable lower layer

Depth h

Barotropic fluid

$L \gg h$

→ shallow water eqn.

BUT: Lower BC now

$$w(x, h_M) = \frac{Dh_m}{Dt}$$

$$= u \frac{\partial h_M}{\partial x}$$

momentum

$$\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u = -\frac{\partial h}{\partial x}$$

(note: $f=0$)

continuity

$$\frac{\partial}{\partial x}\{u(h-h_m)\}=0$$

Linearized set, steady

$\overline{u} \neq 0$ $\quad h' = h - H$

H = mean depth

$h' - h_m$: deviation thickness

$$\bar{U}\frac{\partial}{\partial x}u' + g\frac{\partial h'}{\partial x} = 0$$

$$\bar{U}\frac{\partial}{\partial x}(h' - h_M) + H\frac{\partial u'}{\partial x} = 0$$

solutions (Hatton)

$$c^2 = gH$$

$$h' = \frac{-h_m(\bar{u}^2/c^2)}{1 - \bar{u}^2/c^2}$$

$$u' = \frac{h_m}{H}\left(\frac{\bar{u}}{1 - \bar{u}^2/c^2}\right)$$

$$F_r^2 = \bar{u}^2 / c^2$$

Froude number

$F_r < 1$: subcritical

$F_r > 1$: supercritical

$F_r < 1$: $h_m > 0 \Rightarrow$

$u' > 0$, $h' < 0$

PE $\to$ KE from

$h' < 0$

$F_r > 1$; $h_m > 0$

$u' < 0$, $h' > 0$

$KE \rightarrow PE$

$Fr \sim 1$?

u', h' no longer small

nonlinear

$$u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = 0 \quad (9.39)$$

$$\frac{\partial}{\partial x}\left\{u(h-h_M)\right\} = 0 \quad (9.40)$$

from (9.39):

$$\frac{\partial}{\partial x}\left\{\frac{1}{2}u^2+gh\right\}=0$$

consv of energy

$4 \times (9.39)$

use (9.40) to

eliminate $\frac{\partial}{\partial x} h$

$$\left(1-\frac{u^2}{c^2}\right)\frac{\partial u}{\partial x}=\frac{gu}{c^2}\frac{\partial h_m}{\partial x}$$

Now: $c^2=g(h-h_m)$

where $\frac{\partial h_m}{\partial x}>0 \Rightarrow \frac{\partial u}{\partial x}>0$

Moving up the mountainside, for subcritical flow, $\partial u/\partial x > 0$, so wind speed increases and h decreases. This is consistent with conservation of energy. This opens the potential for Fr approaching 1.

If Fr > 1 after the mountain top, then by the equation on the previous slide, $\partial u/\partial x > 0$, and the flow can go faster and faster down the mountain side, which can make Fr yet more positive. Consistent with energy conservation, h continues to decrease, which makes c decrease further, also making Fr more positive.

Eventually, a hydraulic jump occurs and turbulence is generated, which leads to lee mountain waves.