Geostrophic Adjustment

Energy Considerations

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PE/unit area

$$= \int_0^{\eta'} \rho g z \, dz$$

$$= \frac{1}{2} \rho g (\eta')^2$$

PE/unit area =

$\frac{1}{2} \rho g (x')^2$

$\Delta PE = ?$

infinite in y. Simply
compute ΔPE per unit y

$$\Delta PE = \int_{-\infty}^{+\infty} \rho g \frac{\eta_0^2}{2}\, dx - \int_{-\infty}^{\infty} \frac{1}{2} \rho g (\eta')^2 \, dx$$

$$= 2 \times \int_0^{+\infty} \frac{\rho g \eta_0^2}{2} \left[1 - \left(1 - e^{-x/a} \right)^2 \right] dx$$

$$= \frac{\cancel{2}}{\cancel{2}} \rho g \eta_0^2 \, a \int_0^{\infty} \left[1 - 1 + 2e^{-x/a} - e^{-2x/a} \right] d\frac{x}{a}$$

$$= \frac{1}{2}\rho g \eta_0^2 a \left[+2 - \frac{1}{2} \right]$$

$$\Delta PE = \frac{3}{2}\rho g \eta_0^2 a$$

$$KE\big|_{\text{geostrophic}} = ?$$

per unit area

$$= \int_0^{\eta} \frac{1}{2}\rho (v')^2 \, dz$$

$$KE\big|_g = \frac{1}{2}\rho (v')^2 \eta$$

$KE\big|_g$ per unit y

$$= 2\int_0^{+\infty} \frac{1}{2}\rho (H+\eta')\left(\frac{g\eta_0}{fa}\right) e^{-x/a}\, dx$$

$$KE_g = 2\int_0^{\infty} \frac{1}{2}\rho (v)^2 H \, dx$$

$$= \frac{1}{2}\rho H \left(\frac{g\eta_0}{fa}\right)^2 2\int_0^{\infty} e^{-2x/a}\, dx$$

$$= \frac{1}{2}\rho H \left(\frac{g\eta_0}{fa}\right)^2 \left(\frac{a}{2}\right) 2$$

$$KE\big|_g = \frac{1}{2}\rho g a \eta_0^2$$

$$\Delta PE = 3 \times KE\big|_g$$

$$\left[\text{Note: } a^2 = gH/f^2\right]$$

Transients

$$\frac{\partial^2 \eta'}{\partial t^2} - c^2 \nabla^2 \eta' + f^2 \eta'$$

$$= -f \eta_0 \, \mathrm{sgn}(x)$$

or

$$= -f H \left(2 Q'(x, y, 0) \right)$$

General solution must satisfy I.C.

$$\eta'(t=0) + \eta'_{steady} = \eta_0 \, \mathrm{sgn}(x)$$

$$\eta'(t=0) = -\eta_0 \exp\left\{-\frac{|x|}{a}\right\} \mathrm{sgn}(x)$$

using

$$\eta'_{\text{steady}} = -\eta_0 \operatorname{sgn}(x) + \begin{cases} \eta_0 \exp\{-x/a\} & x>0 \\ -\eta_0 \exp\left\{-\frac{|x|}{a}\right\} & x<0 \end{cases}$$

Equation above (3 slides earlier) can have wavelike solutions:

$$\eta' \propto \exp\{i(kx+ly-\omega t)\}$$

$$\omega^2 = f^2 + (k^2 + l^2) c^2$$

$$c = \pm \sqrt{gH}$$

"Poincaré Waves"