Geostrophic Adjustment

Development

$$\frac{\partial u'}{\partial t} - f v' = -g \frac{\partial \eta'}{\partial x}$$

$$\frac{\partial v'}{\partial t} + f u' = -g \frac{\partial \eta'}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0$$

$$H \times \left\{ \frac{\partial}{\partial x}\left(u' e q\right) + \frac{\partial}{\partial y}\left(v' e q\right) \right\}$$

$$H \frac{\partial}{\partial t}\left\{ \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right\} - fH \left\{ \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y} \right\}$$

$$= -gH \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \eta'$$

$$\frac{\partial^2 \eta'}{\partial t^2} - c^2 \nabla^2 \eta' + fH\zeta' = 0$$

$$c = \pm\sqrt{gH}$$

Take curl of u'-eq. & v'-eq.

$$\frac{\partial}{\partial y}(u'\,\text{eq}) - \frac{\partial}{\partial x}(v'\,\text{eq})$$

$$\frac{\partial}{\partial t}\zeta' + f\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0$$

substitute from continuity

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{f}\right) - \frac{\partial}{\partial A}\left(\frac{n'}{H}\right) = 0$$

$$\frac{\partial}{\partial A}\left(\frac{\rho'}{f} - \frac{n'}{H}\right) = 0$$

$$Q' \equiv \frac{f}{H}\left(\frac{\zeta'}{f} - \frac{\eta'}{H}\right)$$

$$Q' = \frac{\zeta'}{H} - \frac{f}{H^2}\eta'$$

$$Q'(x,y,t) = Q'(x,y,0)$$

Initial state:

$$\zeta'(t=0) = 0$$

$$\eta'(t=0) = -h_0 \operatorname{sgn}(x)$$

$$Q'(t=0) = \frac{+ f h_0}{H^2} \operatorname{sgn}(x)$$

$$\frac{\zeta'(t)}{f} - \frac{\eta'(t)}{H} = -\frac{\eta_0 \operatorname{sgn}(x)}{H}$$

$$fH\zeta' = f^2\eta' + f^2\eta_0 \operatorname{sgn}(x)$$

$$\frac{\partial^2 \eta'}{\partial t^2} - c^2\nabla^2\eta' + f^2\eta' = -f^2\eta_0 \operatorname{sgn}(x)$$

$$= -f t^2 Q'(x,y,0)$$

Is a steady state possible?

$$\frac{\partial}{\partial t}u' = \frac{\partial v'}{\partial t} = 0 \text{ then}$$

$$u' = -\frac{g}{f}\frac{\partial h'}{\partial y} \qquad v' = +\frac{g}{f}\frac{\partial h'}{\partial x}$$

Aside:

$u' \equiv -\frac{\partial \psi}{\partial y}$ & $v' = +\frac{\partial \psi}{\partial x}$

ψ = streamfunction

$$\frac{\zeta'}{f} - \frac{\eta'}{H} = \frac{g}{f^2}\nabla^2\eta' - \frac{\eta'}{H}$$

$$-c^2\nabla^2\eta' + f^2\eta' = -f_0^2\eta_0\,\mathrm{sgn}(x)$$

Note: here $\frac{\partial}{\partial y}() = 0$

physical solution requires

1) finite v'

2) so η' is continuous, esp. at $x=0$

3) η' is finite everywhere

Solve for $x>0$, $x<0$ and match at $x=0$

$$\frac{\eta'}{\eta_0} = \begin{cases} -1 + e^{-x/a} & x > 0 \\ +1 - e^{-|x|/a} & x < 0 \end{cases}$$

$$a = \frac{c}{f} = \frac{\sqrt{gH}}{f}$$

$= $ "λ_R" on other sides

$a = \lambda_R$ = Rossby radius of deformation

$$v' = -\frac{g\eta_0}{fa} \exp\left\{-\frac{|x|}{a}\right\}$$