

$$\frac{\partial^2 \Phi}{\partial z^{*2}} + m^2 \Phi = 0$$

$$m^2 = \frac{N^2}{f_0^2} \left[\frac{\beta}{(u - c_N)} - (k_x^2 + k_y^2) \right] - \frac{1}{4H^2}$$

Solution forms:

$$m^2 > 0$$

$$\Psi = A \exp\{i m t\}$$

when is $m^2 > 0$

$$\frac{N^2}{f^2} \left\{ \frac{\beta}{u} - (k^2 + \ell^2) \right\} - \frac{1}{4H^2} > 0$$

when $C_X = 0$

$$\begin{aligned}
 & \frac{\beta}{\sqrt{1 - (k^2 + l^2)}} > \frac{f_0^2}{N^2} \left(\frac{1}{4H^2} \right) \\
 0 < \sqrt{1 - (k^2 + l^2)} < \beta \left\{ \frac{f_0^2}{N^2} \left(\frac{1}{4H^2} \right) + (k^2 + l^2) \right\}^{-1} \equiv \sqrt{1 - (k^2 + l^2)}
 \end{aligned}$$

$$u_c \uparrow \propto (k^2 + l^2) \downarrow$$

$$m^2 < 0?$$

$$\frac{d^2 \Psi}{dz^2} - m^2 \Psi = 0$$

$$\Psi = A \exp\{\pm |m|z\}$$

Assumed e.g. $|\bar{u}| > |u|$

But $\exp\left\{ + \frac{Z^*}{2H} \right\}$ factor

$\Rightarrow$ nonlinear terms
become important

$$\begin{aligned}
 & \left(\frac{\partial}{\partial x} + \sqrt{1} \frac{\partial}{\partial x} \right) a' + \sqrt{1} \frac{\partial}{\partial y} \\
 & = -\sqrt{1} \frac{\partial a'}{\partial x} - \sqrt{1} \frac{\partial a'}{\partial y}
 \end{aligned}$$

Assume

$$\psi' = N(y, \bar{z}^*) \exp \left\{ i k (x - c_{\bar{x}} t) \right\}$$

$$k c_{\bar{x}} = V \quad \text{or} \quad c_{\bar{x}} = V/k$$

$$\frac{\partial \psi'}{\partial t} = -i k c_{\bar{x}} \psi' = c_{\bar{x}} (i k \psi')$$

$$\frac{\partial \psi'}{\partial t} = -c_x \frac{\partial \psi'}{\partial x}$$

If linear

$$(\bar{u} - c_x) \frac{\partial a'}{\partial x} = -\sqrt{\frac{\partial a}{\partial y}}$$

Linear tf
 $(\bar{u} - c_N) \gg |u'|$

and $d\bar{g}/dy \gg |dg'/dy|$

Breaks down if

$$(\bar{u} - c_k) \Rightarrow 0$$

 $\Rightarrow$ critical surface

Consider how

$$\overline{u} = u(n, z^*)$$

$$u' = e^{ik(x - ct)}$$