Middle Atmosphere

Introduce

$$z^* = -H \ln(p/p_0)$$

p_0 = ref pressure
$\sim$ sfc pressure
$= 1000\,hPa$ typically

H = a scale height

$= RT_s/g$

T_s = a global ave. T

If isothermal

then $\frac{\partial p}{\partial z} = -g\rho = \frac{-gp}{RT_s}$

$\frac{\partial \ln p}{\partial z} = -g/RT_s$

$$\ln\left(p/p_0\right) = -\frac{g}{R T_s}\left(z^* - z_0^*\right)$$

choose $z_0^* = 0$

$$p_0 = p\left(z_0^*\right)$$

$$z^* = -\frac{RT_s}{g} \ln(p/p_s)$$

Note: $p_s = p_s$ in Hotton

$$H = RT_s/g$$

Note:

$$p = p_0 \exp\left\{ \frac{-g z^*}{R T_s} \right\}$$

$$= p_0 \exp\left\{ -z^*/H \right\}$$

Also

$$\rho = p / R T_s$$

$$= \frac{p_0}{R T_s} \exp\left\{ -\frac{z^*}{H} \right\}$$

$$\rho = \rho_s \exp\{ -z^*/H \}$$

Define:

$$\omega^* = D\xi^* \wedge dx$$

Horizontal mom.

p-coordinates:

$$\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u - fv = -\left(\frac{\partial \Phi}{\partial x}\right)_p$$

$$\Phi = gZ$$

If isothermal,

const p sfc is

also const Z^* sfc

$$\frac{D}{Dt}\mathbb{V} + f\hat{k}\times\mathbb{V} = -\nabla_h \Phi$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbb{V}\cdot\nabla_h + \omega^* \frac{\partial}{\partial z}$$

$$\frac{\partial p}{\partial z} = -g\rho$$

$$\frac{\partial z}{\partial p} = -\frac{1}{g\rho} = -\frac{\alpha}{g}$$

$$\frac{\partial z}{\partial p} = -\frac{RT_s}{pg}$$

$$\frac{\partial z}{\partial \ln p} = -RT_s/g$$

$$\frac{\partial \Phi}{\partial \ln p} = -RT_s$$

Note

$$dz^* = -H\,d(\ln p)$$

$$\frac{\partial \bar{\Phi}}{\partial z^*} = +\frac{R T_b}{H}$$

continuity eq.

$$w^* = \frac{Dz^*}{Dt}$$

$$= -H \frac{D}{Dt} \ln p$$

$$= -\frac{H}{\rho}\frac{D}{Dt}\rho$$

$$= -\frac{H}{\rho}\omega$$

$$\frac{\partial \omega}{\delta p} = -\frac{\partial}{\delta p}\left(\frac{p\omega^*}{H}\right)$$

$$= -\frac{p}{H}\frac{\partial}{\delta p}\omega^*$$

$$-\omega^*/H$$

use

$$\frac{\partial \rho}{\partial z^*} = -\frac{1}{H}\rho$$

(because $\rho = \rho_s \exp\left\{-\frac{z}{H}\right\}$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial z^*} = -\frac{1}{H}$$

$$\frac{\partial w}{\partial p} = \frac{1}{\rho_0}\frac{\partial(\rho_0 w^*)}{\partial z}$$

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{1}{\rho_0}\frac{\partial}{\partial z^*}\left(\rho_0 w^*\right) = 0$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\frac{\partial \overline{\Phi}}{\partial z^*} + w^* N^2 = \kappa \overline{J}/H$$

$$N^2 \equiv \left(\frac{R}{H}\right)\left(\frac{\partial \overline{T}}{\partial z^*} + \frac{\kappa \overline{T}}{H}\right)$$

$$\frac{\partial \Phi}{\partial z^*} \doteq -\frac{RT}{H}$$

Also

$$N^2 = \frac{g}{\Theta} \frac{\partial \Theta}{\partial z}$$

$\kappa = R/C_p$

J = diabatic heating