Form

$$\frac{\partial^2 \bar{V}}{\partial y^2} + G\mathbf{V} = 0$$

1) could get waves

2) exponential in y

will require

$\exp\{-(y/L_1)\}$

sidenotes:

1) $h_e \to \infty$

$$\frac{\partial^2}{\partial y^2}\hat{v} + \left[-k^2 - \frac{k}{y}\beta\right]\hat{v} = 0$$

wave solutions
with $\hat{\psi} \sim \exp\{ily\}$
if $\nu = -\beta k/(k^2+l^2)$
or $c = \nu/k$ $c = -\beta/(k^2+l^2)$

2) $\beta = 0$

$$\nu = \pm \left[g h_e \left(k^2 + l^2 \right) \right]^{\frac{1}{2}}$$

$$c = \frac{\nu}{k} = \pm \left[g h_e \left(\frac{k^2 + l^2}{k^2} \right) \right]^{\frac{1}{2}}$$

$$\frac{\partial^2 \vec{V}}{\partial y^2} + G\vec{V} = 0$$

soluton form

$$\bar{V} = \underline{A}\exp\{Ry\}$$

$$r^2 + c_1 = 0$$

$$r = \pm\sqrt{-c_1}$$

if $c_1 < 0 \Rightarrow$ expontial decay

if $c_1 > 0 \Rightarrow$ oscillations in y

General case:

$$\frac{\sqrt{g h_e}}{\beta}\left(-\frac{k}{\nu}\beta - k^2 + \frac{\nu^2}{g h_e}\right) = 2n+1$$

$$n = 0, 1, 2, \ldots$$

(11.37)

Holton

Solutions are equatorially trapped:
use alternate coordinate:

$$\xi = \left(\beta / \sqrt{g h_e}\right)^{1/2} y$$

$$\hat{v}(\xi) = v_1 H_n(\xi) e^{-\xi^2/2}$$

$H_n(\xi)$ Hermite polynomials

$$H_0 = 1;\ H_1(\xi) = 2\xi;\ H_2 = 4\xi^2 - 2$$

$n \neq 0$ 3 solutions:

① eastward prop. gravity wave

② westward gravity wave

③ westward Rossby wave
[equatorially trapped]

$h = 0$?

$$\left(\frac{\nu}{\sqrt{gh_e}} - \frac{\beta}{\nu} - k\right)\left(\frac{\nu}{\sqrt{gh_e}} + k\right) = 0$$

$$\Downarrow$$

quadratic in ν

$$\nu^2 - k\sqrt{gh_e}\,\nu - \beta\sqrt{gh_e} = 0$$

$$\nu = k\sqrt{gh_e}\left\{\frac{1}{2} \pm \left(1 + \frac{4\beta}{k^2\sqrt{gh_e}}\right)^{1/2}\right\}$$

positive root:
eastward prop
inertio-gravity wave

neg. root.
westward
$$\begin{cases} k \to 0 & \text{inertio-grav wave} \\ k \to \infty & \text{Rossby} \end{cases}$$

Kelvin Waves

Special wave with

no meridional motion!

go back to equations with assumed wavelike behavior

$$-i\nu u = -ik\hat{\Phi} \quad (1)$$

$$\beta y \hat{u} = -\partial\hat{\Phi}/\partial y \quad (2)$$

$$-i\nu\hat{\Phi} + gh_e(ik\hat{u}) = 0 \quad (3)$$

Take $\nu \times (1)$

$$-i\nu^2 \hat{u} = -i\nu \hat{\Phi} k$$

$$= -gh_e(k^2)\hat{u}$$

$$\{\nu^2 - gh_e k^2\}\hat{u} = 0$$

or $$c^2 = \frac{\nu^2}{k^2} = gh_e$$

$$c = \pm\sqrt{gh_e}$$

same as gravity waves

Use (1) & (2) to eliminate $\hat{\Phi}$ again

$$-i\nu \frac{\partial \hat{u}}{\partial y} = ik \frac{\partial}{\partial y} \hat{\Phi}$$

$$= (ik) \beta y \hat{u}$$

$$\frac{\partial}{\partial y} \hat{u} + (\beta y / c) \hat{u} = 0$$

$$\beta y = -c \frac{\partial}{\partial y} \ln \hat{u}$$

integrate in y

$$\frac{\beta y^2}{2} = -c \ln(\hat{u}) \Big|_{y=0}^{y=y}$$

$$\beta y^2/2 = -c \ln\left[\frac{\hat{u}(y)}{\hat{u}(0)}\right]$$

$$\hat{u}(0) = u_0$$

$$\hat{u} = \hat{u}_0 \exp\left\{-\frac{\beta y^2}{2c}\right\}$$

<u>Need $c>0$</u>

only positive roots valid

e-folding scale

$$\frac{\beta y_{er}^2}{2c} = 1 \Rightarrow y_{er} = \sqrt{\frac{2c}{\beta}}$$