

Propagation of
Energy
Group Velocity

$$\vec{C}_g = \hat{r} \frac{\partial}{\partial k} (k c) + \hat{\theta} \frac{\partial}{\partial \alpha} (l c)$$

k = Zonal wave number

l = meridional "
 c = phase speed

$$\vec{C}_g = i \left(k_x C + C \right)$$

for $l \ll 0$

for simple gravity
waves

$$C = \bar{u} \pm \sqrt{gh}$$

$\Downarrow$

$$c_g = \lambda c$$

For Rossby waves

$$C \approx \bar{u} - \beta/k^2$$

$$\begin{aligned} c_g &= \lambda \left(k \frac{dC}{dk} + C \right) \\ &= \lambda \left(+k \frac{2\beta}{k^3} + \bar{u} - \beta/k^2 \right) \end{aligned}$$

$$\vec{C}_g = i(\vec{U} + \beta/k^2)$$

$$\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$- \beta y u = g \frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial x} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \beta y u = -g \frac{\partial h}{\partial y}$$

$$\frac{\partial}{\partial x}(h-h_B) + \frac{\partial}{\partial x}(u(h-h_B))$$

$$+ \frac{\partial}{\partial y}(v(h-h_B)) = 0$$

Assume

1) $f = \beta M$ $y=0$ is the equator

2) Basic State: ✓
motionless, flat bottom
($\Rightarrow h_b = 0$), depth = h_e

perturbations:

$$u = u', v = v'$$

Holtan uses $\Phi = g_h$

$$\frac{\partial u'}{\partial t} - \beta_y v' = -\frac{\partial \Phi'}{\partial x}$$

$$\frac{\partial v'}{\partial t} + \beta_y u' = -\frac{\partial \Phi'}{\partial y}$$

$$\frac{\partial \Phi'}{\partial t} + g h_c \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = 0$$

$$\begin{pmatrix} u' \\ v' \\ w' \end{pmatrix} = \begin{bmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{h}(y) \end{bmatrix} e^{i(Kx - \omega t)}$$

$\omega = \text{frequency}$
 $C = \omega / K \quad \text{or} \quad \omega = K C$

So $\frac{\partial}{\partial x} u' = +ik u'$

$$\frac{\partial}{\partial y} u' = \frac{\partial}{\partial y} u e^{i(kx - \omega t)}$$

but $\frac{\partial}{\partial x} u' = ik u e^{i(kx - \omega t)}$

$$\frac{\partial u}{\partial t} = -iV u e^{i(kx - \omega t)}$$

Similarly for v, h

$$\textcircled{1} -i\nabla\hat{u} - \beta y \hat{u} = -ik \hat{\Phi}$$

$$\textcircled{2} -i\nabla\hat{u} + \beta y \hat{u} = -\frac{\partial}{\partial y} \hat{\Phi}$$

$$\textcircled{3} -i\nabla\hat{\Phi} + g h_e \left(k \hat{u} + \frac{\partial \hat{u}}{\partial y} \right) = 0$$

Note: $\hat{u} = \frac{i\beta y}{\nu} \hat{u} + \frac{k}{\nu} \hat{\Phi}$

Substitute into (2)

$$-i\cancel{V}V_4 + i\beta_{\cancel{y}}^2 \cancel{V} + \frac{\beta_{\cancel{y}} k}{\cancel{V}} \Phi$$

$$\left(\beta_{\cancel{y}}^2 - \cancel{V}^2 \right) \cancel{V} = - \frac{\cancel{V}}{\cancel{V}} \Phi$$

$$[multiplied by -i\cancel{V}]$$

substitute into (3)

$$-i) \hat{\Phi} + g h e \left(-\frac{k}{v} \beta_M \hat{v} + i k^2 \hat{\Phi} \right) + g h e \frac{\partial \hat{v}}{\partial y}$$

multiply by (iv)

$$(V^2 - K^2 g_{he}) \frac{\hat{\Phi}}{\hat{\Lambda}} +$$

$$iV g_{he} \left(\frac{\partial V}{\partial y} - \frac{K}{V} \beta y \hat{V} \right)$$

$$= 0$$

[Ho/Ho] $\approx$ (11.34) $\pm$ (11.35)

eliminate Φ

$$\frac{\partial}{\partial y} \left[\frac{1}{2} v^2 + \left(\frac{v^2}{g h_e} - K - \frac{K}{D} \beta \right) - \frac{\beta^2 y^2}{g h_e} \right] \uparrow = 0$$