Thompson’s Method

The subtle details and results

Thompson's 2 "tricks"

Unknowns are

$v, \frac{\partial}{\partial x}u, \frac{\partial}{\partial x}h$

$$\frac{\partial}{\partial t} u' = -ikcu'$$

$$= -c(ik)u'$$

$$= -c\frac{\partial u'}{\partial x}$$

$$\frac{\partial^2 v'}{\partial y^2} = -k^2 v'$$

$$(\bar{u} - c)\frac{\partial u'}{\partial y} - f v' + g\frac{\partial h'}{\partial x} = 0$$

$$-k^2(\bar{u} - c) v' + \beta v' + f\frac{\partial u'}{\partial x} = 0$$

$$(\bar{u} - c)\frac{\partial h'}{\partial x} - \frac{f\bar{u}}{g} v' + \bar{h}\frac{\partial u'}{\partial x} = 0$$

$$\begin{vmatrix} \beta - k^2(\bar{u}-c) & 0 & f \\ -f & g & (\bar{u}-c) \\ -\frac{f\bar{u}}{g} & (\bar{u}-c) & \bar{h} \end{vmatrix} = 0$$

$$\left[\beta - k^2(\bar{u}-c)\right]\left[gh - (\bar{u}-c)^2\right]$$

$$- f^2\left[(\bar{u}-c) - \bar{u}\right] = 0$$

Assume $f = 0$

$$\left(\frac{\beta}{k^2} - (\bar{u}-c)\right)\left[gh - (\bar{u}-c)^2\right] = 0$$

2 possible solutions:

$$c - \bar{u} = -\beta / k^2$$

$$c - \bar{u} = \pm\sqrt{g\bar{h}}$$

1. Assume

$$(\bar{u} - c)^2 \ll g\bar{h}$$

then

$$c - \bar{u} = -\frac{\beta + f^2\bar{u}/g\bar{h}}{K^2 + f^2/g\bar{h}}$$

② $|\bar{u}-c| >> \bar{u}$

$$(\bar{u}-c)^2 = g\bar{h} + f^2/k^2$$

or $(c-\bar{u}) = \pm\sqrt{g\bar{h} + f^2/k^2}$