$$w = \frac{dz}{dt}$$

$$= \frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} + v\frac{\partial z}{\partial y}$$

for $z = h_B$

$$w(h_B) = u\frac{\partial h_B}{\partial x} + v\frac{\partial h_B}{\partial y}$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)(z - h_B) = w(h_B) - w(z)$$

$$\left(\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v\right)(z - h_B) = u\frac{\partial}{\partial x}h_B + v\frac{\partial}{\partial y}h_B - w(z)$$

$$w = \frac{dz}{dt} \text{ and at } z = h$$

$$w(h) = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y}$$

$$\left(\frac{\partial}{\partial x}u+\frac{\partial v}{\partial y}\right)\left(h-h_B\right)=$$

$$u\frac{\partial}{\partial x}h_B+v\frac{\partial}{\partial y}h_B$$

$$-\frac{\partial h}{\partial t}-u\frac{\partial h}{\partial x}-v\frac{\partial h}{\partial y}$$

$$\frac{\partial}{\partial t} h + \left(\frac{\partial}{\partial x} u + \frac{\partial v}{\partial y} \right)(h - h_B) + u \frac{\partial}{\partial x}(h - h_B) + v \frac{\partial}{\partial y}(h - h_B) = 0$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\left[u(h-h_B)\right] + \frac{\partial}{\partial y}\left[v(h-h_B)\right] = 0$$

Note: $\partial h_B / \partial t = 0$

$$\frac{\partial}{\partial t}(h-h_B) + \frac{\partial}{\partial x}\left[u(h-h_B)\right] + \frac{\partial}{\partial y}\left[v(h-h_B)\right] = 0$$

Back to Thompson (1961)

Vorticity $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

$$\frac{\partial}{\partial t} u + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$- f v + g \frac{\partial h}{\partial x} = 0$$

$$\frac{\partial}{\partial t} v + u \frac{\partial}{\partial x} v + v \frac{\partial}{\partial y} v + f u + g \frac{\partial h}{\partial y} = 0$$

for $\int$, $\frac{\partial}{\partial y}$ of u eq.

$+ \frac{\partial}{\partial x} + v$ eq

$$\frac{\partial}{\partial t}\zeta + u\frac{\partial}{\partial x}\zeta + v\frac{\partial}{\partial y}\zeta + v\frac{\partial f}{\partial y} + (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Assume $u = \bar{u} + u'$

$v = v'$

$\bar{u} = \text{constant}$

$\zeta = \zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}$

Basic state has

$$f\bar{u} = -g\frac{\partial\bar{h}}{\partial y}$$

$$\bar{h} = \bar{h}(y)$$

$$\left(\frac{\partial}{\partial t}+\bar{U}\frac{\partial}{\partial x}\right)u'-fv'$$

$$+g\frac{\partial}{\partial x}h'=0$$

($u'\frac{\partial}{\partial x}u'$ is <u>small</u>.)

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\frac{\partial v'}{\partial x}-\frac{\partial u'}{\partial y}\right)+\beta v'+f\left(\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}\right)$$

$$=\partial$$

Assume wavelike
behavior in x
nothing in y

$$\frac{\partial}{\partial t}h' + \bar{u}\frac{\partial h'}{\partial x} + v'\frac{\partial \bar{h}}{\partial y} + \bar{h}\frac{\partial u'}{\partial x} = 0$$

3 equations, 3 unknowns

u', v', h'

Assume wave structure

$$u' = A_u \exp\{ik(x-ct)\}$$

$$\begin{Bmatrix} u' \\ v' \\ h' \end{Bmatrix} = \underbrace{\begin{Bmatrix} A_u \\ A_v \\ A_h \end{Bmatrix}}_{\text{const.}} e^{ik(x-ct)}$$

$$e^{ix} = \cos x + i \sin x$$

$$\frac{\partial}{\partial x} e^{ik(x-ct)} = ik\, e^{ik(x-ct)}$$

$$\frac{\partial}{\partial t} e^{ik(x-ct)} = -ick\, e^{ik(x-ct)}$$

Here "k" = "α"

$$k = 2\pi/L$$

so, e.g.

$$\frac{\partial}{\partial x}u' = iku; \quad \frac{\partial}{\partial t}h' = -ickh'$$

$$(\bar{u}-c)iku' - fv' + gikh' = 0$$

$$(\bar{u}-c)(ik)(ik)v' + \beta v' + fiku' = 0$$

$$(\bar{u}-c)(ik)h' + v'\frac{f}{g}\bar{u} + ik\bar{h}u' = 0$$