# MT454 - Energy Cycle

Two-layer, Q-G Model

$APE \propto \overline{\theta'^2}$

What is it in 2-Layer model?

Focus on wave part +

Suggest

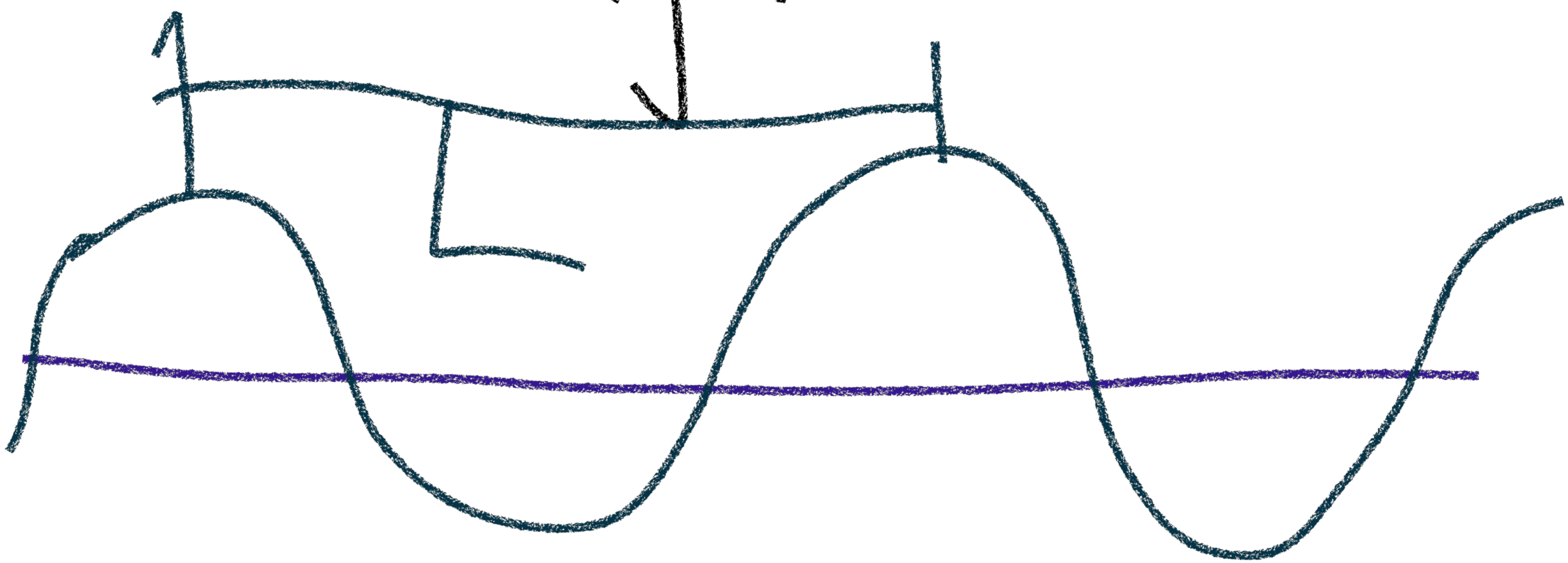
$$(\psi_1' - \psi_3')^2 \propto \frac{\text{APE in wave}}{\text{part of flow}}$$

Zonal average:

$$[X] = \frac{1}{2\pi}\int_0^{2\pi} X(\phi)\, d\phi$$

$\phi$ = longitude

for wave, can average over 1 wavelength



$$\langle \chi \rangle = \frac{1}{L}\int_0^L \chi \, dx$$

Note:

$$\left\langle \frac{\partial}{\partial x} \chi \right\rangle = \frac{1}{L}\left\{ \chi(L) - \chi(0) \right\}$$

$$= 0$$

Why average?

1) Simplifies

2) so easier to understand

3) interested in <u>net</u> behavior

# Procedure

1) Use linear equations

2) (Level 1 vorticity) $\times \psi_1'$

3) same for $\psi_3$

& $(\psi_1' - \psi_3')$

Thus, e.g.,

$$\left\langle \psi_1' \left\{ \left( \frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x} \right) \frac{\partial^2 \psi_1'}{\partial x^2} + \beta \frac{\partial \psi_1'}{\partial x} = \frac{f_0}{\Delta p} \omega_2' \right\} \right\rangle$$

then get, e.g.

$$\left\langle \psi_1' \left( \frac{\partial}{\partial t} \frac{\partial^2 \psi_1'}{\partial x^2} \right) \right\rangle$$

$$= -\frac{1}{2} \left\langle \frac{\partial}{\partial t} \left( \frac{\partial \psi_1'}{\partial x} \right)^2 \right\rangle$$

then

$$\frac{1}{2}\left\langle \frac{\partial}{\partial t}\left(\frac{\partial \psi_1'}{\partial x}\right)^2 \right\rangle$$

$$= -\frac{f_0}{\Delta p}\langle \omega_2' \psi_1' \rangle$$

$$\frac{1}{2}\left\langle \frac{\partial}{\partial t}\left(\frac{\partial \psi_3'}{\partial x_i}\right)^2 \right\rangle$$

$$= +\frac{f}{\Delta \rho}\langle \omega_2' \psi_3' \rangle$$

$$
\frac{1}{2}\left\langle \frac{\partial}{\partial t}\left(\psi_1' - \psi_3'\right)^2 \right\rangle =
$$

$$
= U_T \left\langle \left(\psi_1' - \psi_3'\right) \frac{\partial}{\partial x}\left(\psi_1' + \psi_3'\right) \right\rangle
$$

$$
+ \frac{\sigma \Delta p}{f_0} \left\langle \omega_2' \left(\psi_1' - \psi_3'\right) \right\rangle
$$

$$u_T = (u_1 - u_3)/2$$

$$\left(\frac{\partial}{\partial x} y_j\right) = (v_j')^2$$

$$K' \equiv \frac{1}{2}\left\langle\left(\frac{\partial \psi_1'}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial \psi_3'}{\partial p}\right)^2\right\rangle$$

$$\frac{d}{dt}K' = -\frac{f_0}{\Delta p}\left\langle \omega_2'\left(\psi_1' - \psi_3'\right)\right\rangle$$

$$P' \equiv \frac{\lambda_\Delta^2}{2}\left\langle\left(\psi_1' - \chi_3'\right)^2\right\rangle$$

$$\lambda^2 = f_0^2/\sigma(\Delta p)^2$$

$$\frac{d}{dt} p' = \lambda^2 u_T \times \left\langle (\psi_1' - \psi_3') \frac{\partial}{\partial x} (\psi_1' + \psi_3') \right\rangle + \frac{f_0}{\Delta p} \left\langle \omega_2' (\psi_1' - \psi_3') \right\rangle$$

K' increase?

$$\langle \omega_2'(\psi_1' - \psi_3') \rangle < 0$$

need $\omega_2' < 0$ where $(\psi_1' - \psi_3') > 0$

[upward] [warm]

and

$\omega_2' > 0$ where $(\psi_1' - \psi_3') < 0$

[sinking] [cool]

- denser air sinking
- center of mass ↓
- $P \rightarrow K$

other term in $\frac{d}{dt} p'$

need a positive correlation of 500 hPa thickness and 500 hPa meridional wind

Need alignment of temp & $v$ waves: