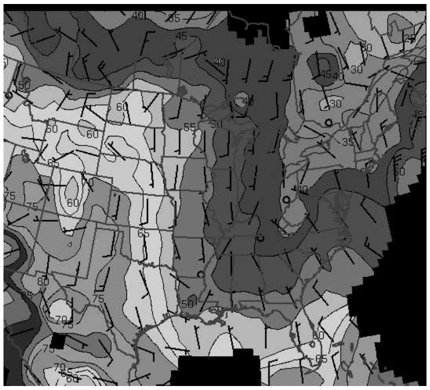
Exam 1 - Meteor. 454 (Holton, Chapter 5)

There are three questions, with point values as marked (100 pt. total).

Time allowed: 50 minutes



1. Recently, a storm in the northeastern U.S. caused relatively cold air to blow over the nearby ocean. The air at 2 meters had a temperature of about 45 ˚F = 7 ˚C. The ocean temperature in the same area was 12 ˚C.

(a) [10 points] Why did a well-mixed (unstable) PBL develop?
Air temps colder than ocean ➔ heating of air near surface, creating unstable stratification. Ocean temps persist due to high ocean heat capacity.

(b) [10 points] A bulk aerodynamic formula can be used to compute the surface flux of sensible heat just as we did in class for momentum fluxes. In this case, the formula is

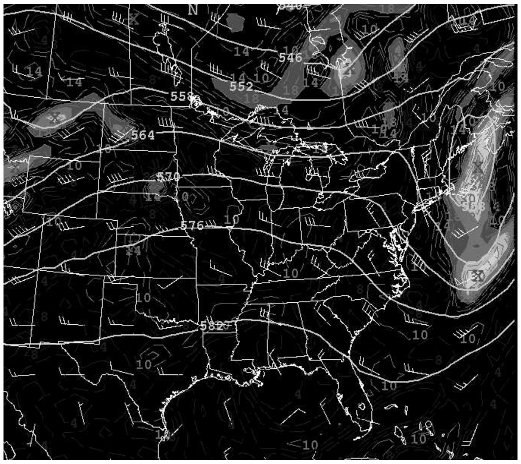
$$\left(\overline{w'\theta'}\right)_{sfc} = C_d\left|\vec{V}_s\right|\left(\theta_s - \theta_a\right) ,$$

where θ_s is the temperature of the underlying surface, θ_a is the 2-m air temperature, and the flux is positive flux is upward. (You can assume the surface pressure is ≈ 1000 mb.) Suppose $C_d = 10^{-3}$ and $\left|\vec{V}_s\right| = 5$ m/s. What is the heat flux into the atmosphere for the location above? Be sure to state your units!

ANSWER: $\left(\overline{w'\theta'}\right)_{sfc} = 10^{-3} \times 5\text{x}(285 - 280) = 25 \times 10^{-3}\ m - \text{deg/sec}$

(c) [10 points] Potential temperature here increases at a rate of 3 K/km from the surface value (7 ˚C = 280 K). What eventually will be the depth of the PBL as the ocean warms the air above it?
PBL extends up to where parcel with potential temp of surface no longer has positive buoyancy vs. initial atmospheric profile. Thus, PBL top is at level where $\theta_{air} = \theta_s =$ 285 K. PBL top = (285-280)/3 = 5/3 km = 1.67 km

(d) [10 points] If the relationship between w' and θ' given above is typical for all levels in the PBL, how is the center of mass of the PBL changing? Have warm air rising (air warmed near the surface) and cooler air sinking ➔ lowering of center of mass.



2. The same storm system producing the conditions above is forecast to produce the cut-off low that appears in the 500 hPa height field to the left, along the East Coast. The same pattern will appear at 850 hPa and 700 hPa, so we can analyze the flow as barotropic. The vorticity of the cut-off low will be ≈ 3 x 10^{-5} s^{-1}. For this location, you can assume f = 1 x 10^{-4} s^{-1}, the depth of the troposphere is 10 km and the boundary layer at the forecast time is approximated by an Ekman layer.

(a) [10 points] Suppose the eddy diffusivity is K_m = 5.0 m^2/s. What will be the vorticity of this cut-off low in 5 days? Need damping time for spin-down of a low over an Ekman layer. This is given in the equations below:

$$\tau_e = H\left|2/fK_m\right|^{1/2} = 10\times10^3\sqrt{\frac{2}{10^{-4}\times5}} = 6.32\times10^5\ \sec = 7.3\ days$$

. Then we need the equation for how vorticity evolves with time in this situation: $\zeta(t) = \zeta(0)\exp\{-t/\tau_e\}$ = $3\text{x}10^{-5}$ exp{-5/7.3} =$1.5\text{x}10^{-5}$ $\sec^{-1}$.

(b) [10 points] Sketch the primary and secondary circulations around this low center at some level in the *free atmosphere*.
(See CoursePack notes, pages 43 and 48.)

(c) [10 points] What is the initial vertical wind speed at the top of the Ekman layer?
Use the Ekman pumping formula (from below):

$$w = \varsigma_g\left|K_m/2f\right|^{1/2}\frac{f}{|f|} = 3\times10^{-5}\left|\frac{5}{2\times10^{-4}}\right|^{1/2}\times\frac{1\times10^{-4}}{\left|1\times10^{-4}\right|} = 4.7\times10^{-3}\,m/\sec = 4.7\ mm/\sec$$

(d) [10 points] Suppose instead the eddy diffusivity is K_m = 2.5 m^2/s. How long will it take to reduce the vorticity to the same magnitude as in part (a)?
We want the τ_e for this K_m and then the time t needed to reduce the vorticity by about half, from $3\text{x}10^{-5}$ to $1.5\text{x}10^{-5}$ $\sec^{-1}$. What this means is that we want the same exponent for the "exp" function. Before it was - 5/7.3. The new "t" will be such that $t_{new} = (\tau_e)_{new}$ x (5/7.3). Now we have

$$(\tau_e)_{new} = H\left|2/f(K_m)_{new}\right|^{1/2} = 10\times10^3\sqrt{\frac{2}{10^{-4}\times2.5}} = 8.94\times10^5\ \sec = 10.4\ days$$

Thus, $t_{new} = (\tau_e)_{new}$ x (5/7.3) = 7.1 days

We did not do this material in class. Nonetheless, if you want the answers …

3. A surface layer was observed at a one of the Iowa State experimental farms. Wind measurements at two levels showed:

z [m]	$\overline{U}$ [m/s]
0.20	0.0
10.0	8.0

(a) [10 pt] What is the roughness length at this location?

Roughness length = z_o = 0.20 meters

(b) [10 pt] What is the surface turbulent momentum flux, $\left(\overline{u'w'}\right)_s$?

Use $\bar{u} = (u_* / k_K)\ln(z/z_o) \Rightarrow u_* = \left(\bar{u}\right)k_K \big/ \ln\left(z/z_o\right)$. Then use u(10m) = 8 m/s, so that

$$u_* = \left(\bar{u}\right)k_K \big/ \ln\left(z/z_o\right) = 8 \times 0.4 \div \ln\left(\frac{10}{0.2}\right) = 0.82\ m/s$$

and then $\left|w'u'\right|_s = u_*^2 = 0.67\ m^2/s^2$.

<u>Some random equations</u>

$\gamma = \left(f/2K_m\right)^{1/2}$ $\qquad u_*^2 = \left|w'u'\right|_s$ $\qquad d = \pi\left(2K_m/f\right)^{1/2}$

$\bar{u} = (u_*/k_K)\ln(z/z_o)$ $\qquad w = \varsigma_g \left|K_m/2f\right|^{1/2} \dfrac{f}{|f|}$ $\qquad \theta = T\left(\dfrac{p_o}{p}\right)^{R/C_p}$

$\tau_e = H\left|2/fK_m\right|^{1/2}$ $\qquad \dfrac{\overline{D}}{Dt}\bar{\theta} = -\bar{w}\dfrac{\partial \theta_o}{\partial z} - \left\{\dfrac{\partial}{\partial x}\left(\overline{u'\theta'}\right) + \dfrac{\partial}{\partial y}\left(\overline{v'\theta'}\right) + \dfrac{\partial}{\partial z}\left(\overline{w'\theta'}\right)\right\}$

$\left(\overline{w'\theta'}\right) = -K_h \dfrac{d\bar{\theta}}{dz}$ $\qquad d = \pi\left(2H\zeta_g/f\right)^{1/2}$ $\qquad BPL = \overline{w'\theta'}(g/\theta_o)$

$\left(\overline{w'u'}\right) = -K_m \dfrac{d\bar{u}}{dz}$ $\qquad K_m = \overline{(\xi')^2}\left|\dfrac{\partial \vec{V}}{\partial z}\right|$ $\qquad k_K \approx 0.4$