Atmospheric Waves

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ABSTRACT

Understanding synoptic waves allows meteorologists to accurately forecast weather patterns. This study examines upper-level atmospheric motion in waves and winds from September 11, 2011 to November 11, 2011. It includes data regarding wave number, amplitude, wave motion, and wind speed, which were found by observing atmospheric waves at 500 hectopascal (hPa) and the 150-300 hPa layer. The wave characteristics were analyzed and plotted against one another and compared to a generally accepted dynamic theory, the Rossby Wave Theory. Overall, the results varied in their agreement with the Rossby Wave Theory, which may be a result of the analysis procedure, short period of analysis, abnormalities from average within the domain, and/or the assumptions made from using the Rossby Wave Theory.

1. Introduction

Large-scale waves are a crucial part of the forecasting process because they influence the synoptic weather patterns across the globe. These waves average to have zonal flow, but small fluctuations in this flow are common. These fluctuations are known as troughs and ridges, which form the wave pattern. A long wave is a wave with a wavelength greater than 1000 kilometers (km). Troughs and ridges cause changes in atmospheric variables, such as vorticity and temperature. For example, dynamic forcing occurs ahead of a trough, which induces upward motion. This creates a surface low-pressure system ahead of the trough, which often results in precipitation and temperature changes at the surface. From this example, one can see that troughs and ridges influence the weather. Smaller variations within large-scale waves can affect the weather as well. These waves are commonly known as short waves. Short waves affect a region of roughly 100-1000 km. These waves create upper-level forcing, which often results in precipitation. The precipitation with short waves is confined to a smaller area than compared to long waves.

While waves fluctuate daily to create long waves and short waves, they also change seasonally. In fact, waves vary throughout the year in strength and quantity. During the spring, waves tend to weaken, while they strengthen in intensity during the fall. The wave number during these months is generally greater than those in summer and winter.

Important characteristics of the large-scale flow include amplitude, wave number, and wave motion. Amplitude describes how much energy a wave contains; therefore, larger amplitude corresponds to more energy. Wave number describes how many trough-ridge patterns are present. The Rossby Wave Theory gives insight into wave motion. The phase speed, which is the magnitude of wave motion, is related to the zonal wind. Phase speed is expressed as

\[ C_x = \frac{v}{k} = u - \left\{ \frac{\beta}{(k^2 + l^2)} \right\} \]

where \( C_x \) is the phase speed in the x-direction, \( u \) is the zonal wind, \( \beta \) describes how planetary vorticity varies with latitude, \( k \) is the wave number in the x-direction, and \( l \) is the wave number in the y-direction. Thus, as the wave number decreases, the phase speed increases. Also, as the positive zonal wind increases, the phase speed increases in the positive x-direction. If there is no zonal flow, the wave will propagate to the west, which is in the negative x-direction. This is
observed in reality. The zonal wind is usually small above a cutoff low-pressure system because the system is separated from the main jet stream. Observations show that these systems can actually propagate westward. For non-cutoff systems, the zonal flow is usually large enough so the wave propagates east. Given an eastward zonal flow, longer waves propagate slower eastward than shorter waves, because the wave number is greater for longer waves.

Three hypotheses were formed at the beginning of the study: 1) A larger wave number implies a lower amplitude, 2) A larger wave number implies a faster wave motion, and 3) Faster upper-level winds implies faster wave motion.

2. Data and Methods

a. Waves

Waves data was taken from the Iowa State Weather Products website from September 11, 2011 to November 11, 2011. The products used to conduct the analysis were the Northern and Southern Hemisphere 500 hPa analyses and the global average zonal wind. Analyses of the Northern and Southern Hemisphere 500 hPa flows were centered on 50° latitude. The data collection included wave number, average amplitude, and average wave motion. The wave number, k, was equal to half the number of times the 5580 m height contour intersected the 50° latitude line. This study defined the average amplitude, A, as

\[ A = \frac{(A_R + A_T)}{2}, \]

where \( A_R \) was the average ridge amplitude and \( A_T \) is the average trough amplitude.

\[ A_R = \frac{(a_{R1} + a_{R2} + \ldots + a_{Rk})}{k} \]

\[ A_T = \frac{(a_{T1} + a_{T2} + \ldots + a_{Tk})}{k} \]

where \( a_{R1}, a_{R2}, \) and \( a_{Rk} \) were the individual ridge amplitudes and \( a_{T1}, a_{T2}, \) and \( a_{Tk} \) were the individual trough amplitudes. The maximum distance the 5580 m height contour extended below or above the 50° latitude line was the amplitude of the trough or ridge, respectively. For the Southern Hemisphere, the 5580 m height contour was not used to determine wave number and amplitude; rather, the 5280 m contour was used.

Wave motion, C, was calculated by averaging the distance two individual waves traveled during a two day period.

\[ C = \frac{(LON(day+1) - LON(day-1))}{2} \]

The wave motion for a particular day was calculated by examining the distance a wave traveled from its starting point the previous day to its location the following day. The distance the wave traveled was measured in degrees of longitude.

b. Zonal Wind

Data collection for zonal wind consisted of determining the average speeds at 500 hPa and maximum speeds for the 150-300 hPa layer in both the Northern and Southern Hemispheres. The wind speed was determined at 50° latitude, because the jet stream was in close proximity to
this latitude. Data was recorded from September 11, 2011 to November 11, 2011. Positive values of zonal wind indicated an eastward wind, while negative values indicated a westward wind.

c. Accuracy and Limitations

Every variable that was calculated had limitations. If the 5580 m or 5280 m height contour only reached the 50° latitude line, it was excluded as a wave. For example, there may have been five distinct trough-ridge patterns present in the 500 hPa flow, but some of the troughs or ridges may not have been deep or high enough to cross the 50° latitude line. This would have caused the wave number to be less than what was actually occurring in the wave pattern. Closed pressure systems were another limitation. If a closed pressure system was below the 50° latitude line, with the 5580/5280 height contour at 50° latitude or higher, this system was not counted (Figure 1).

If a closed pressure system was above the 50° latitude line with the 5580/5280 height contour below the 50° latitude line, this system was counted. In this case, the amplitude was quantified as the lowest height that connected to the pole. From Figure 2, an amplitude of 300 m would have been calculated for this wave using the above procedure.

In addition to limitations in calculating the amplitude, there were limitations in calculating the wave motion. Waves were chosen based on how they moved and how they developed. Waves that moved slowly or retrograded were not used to calculate wave motion. Also, waves that weakened significantly were ignored.

Finally, there were limitations in calculating zonal winds. The 50° latitude line was estimated in each hemisphere due to the lack of clear markings, so the magnitude of the zonal winds may contain error. Occasionally a strong gradient was present at 500 hPa or within the 150-300 hPa layer, which made it difficult to determine an accurate wind value. In some locations winds were weak or even slightly negative. Because sign indicates direction, this study took the absolute value to be the maximum wind speed. For example, -10 m/s was considered greater than 5 m/s, and thus, the maximum wind speed would have been -10 m/s.
3. Results

a. Zonal Wind

Zonal wind influences the speed at which waves propagate. An eastward zonal wind will cause a wave to move eastward, while a westward zonal wind will cause a wave to move westward. Zonal wind differs between the Northern and Southern Hemispheres.

![500 hPa Zonal Wind vs. Date](image)

Figure 3

Overall, the Southern Hemisphere (SH) 500 hPa zonal wind (U500) is generally greater in magnitude than the Northern Hemisphere (NH) 500 hPa zonal wind during the period (Figure 3). The average 500 hPa zonal wind for the Southern Hemisphere is 14.33 m/s while the average for the Northern Hemisphere is 10.65 m/s. This may be due to the extra land mass in the Northern Hemisphere at 50°N, which would likely decrease the wind speed from frictional effects, even at 500 hPa (due to high topography). The Southern Hemisphere has less landmass at 50°S. In fact, the only landmass that crosses 50°S is the southern tip of South America, which leads to less frictional dissipation.

The range of the 500 hPa zonal wind for the Southern Hemisphere is generally larger than the Northern Hemisphere as well. The data ranges from -15 m/s to 45 m/s in the Southern Hemisphere, while only from -8 m/s to 21 m/s in the Northern Hemisphere, which explains the low coefficients of determination (R² values), especially for the Southern Hemisphere. The wider range in the Southern Hemisphere is also likely due to frictional effects. For the Northern Hemisphere, the larger landmass would cause less variability in the winds because friction would decrease the wind speed of faster winds more than for slower winds. In the Southern
Hemisphere, there would be less frictional effects so faster winds would remain faster than the slower winds.

Over the domain the Southern Hemisphere winds slightly increase (Figure 3). This agrees with our mindset. As the Southern Hemisphere transitions from winter to spring, the stronger temperature gradient will cause stronger winds. The Northern Hemisphere trend opposes that of the Southern Hemisphere, as winds slightly decrease over the domain. This also agrees with our mindset. As the Northern Hemisphere transitions from summer to fall, the temperature gradient at the beginning of this transition changes minimally or slightly decreases, which causes a slight decrease in the 500 hPa zonal wind. By the end of this transition period, the temperature gradient increases, and the zonal wind increases. This is observed in Figure 3.

There is an opposing relationship between the 500 hPa zonal wind and wave amplitude in the Northern Hemisphere.

![500 hPa Zonal Wind vs. Amplitude NH](image)

The wave amplitude generally increases as the 500 hPa zonal wind decreases (Figure 4). This occurs on both short time scales (roughly a week) and for the overall pattern. Notice that as the 500 hPa zonal wind increases, the wave amplitude decreases for most of the short time periods. There are a few times when this pattern disappears, specifically at the end of the period, but the general pattern seems to hold true. So, it seems that as the 500 hPa zonal wind increases, the amplitude decreases. This is explained using the Rossby Wave Theory. A slower zonal wind
corresponds to a slower wave motion, which in return, corresponds to a smaller wave number, thus producing a greater wave amplitude. In addition, the magnitude of change seems to be similar. As the wave amplitude increases, the 500 hPa zonal wind decreases by around the same amount, relative to the perspective scales. Also, as the wave amplitude decreases, the 500 hPa zonal wind increases by around the same amount.

The Southern Hemisphere shows the opposite pattern.

Figure 5

The wave amplitude generally increases as the 500 hPa zonal wind increases (Figure 5). Disagreement also exists when looking at the smaller scale. For the Southern Hemisphere, as the wave amplitude increases, the 500 hPa zonal wind generally increases as well. This is particularly apparent during the time period from September 19 until October 2. However, this correlation is less prominent compared to the Northern Hemisphere’s correlation.

Both the wave amplitude and 500 hPa zonal wind slightly increase during the overall period (Figure 5). This disagrees with theory. It is expected that as the 500 hPa zonal wind increases, the wave amplitude decreases. One possible explanation for this disagreement is the low correlation. The 500 hPa zonal wind only slightly increases over the domain, with a slope of only 0.0177. Also, the coefficient of determination is small for this data (smaller than the Northern Hemisphere data), meaning that the trend is weaker than that of the Northern Hemisphere. Possible human error when calculating the values of 500 hPa zonal wind could cause the weak slope of the zonal wind to become negative. Then the observations would agree with theory.
b. Wave Amplitude

Wave amplitude varies over a temporal scale and generally increases with time in both the Northern and Southern Hemispheres (Figure 6 and Figure 7). This is due to the wave number decreasing during the time period (Figure 9 and Figure 10). As the wave number decreases, the amplitude generally increases.

![Amplitude vs. Date NH](image)

Figure 6

![Amplitude vs. Date SH](image)

Figure 7

Also, the range of the amplitudes increases through the domain for the Southern Hemisphere. In the Southern Hemisphere, the range of values begins from 129 m to 215 m, and
ends with a range from 165 m to 295 m. Notice that the Southern Hemisphere’s range increases from 86 m to 130 m. In the Northern Hemisphere, the range of values begins from 110 m to 190 m and ends with a range from 162 m to 248 m, so the range stays roughly constant.

The zero values of the amplitude for the Southern Hemisphere represent days with no waves present, using the study’s analysis procedure. There may have been waves that the analysis procedure did not recognize, so the data shows no waves for that day and thus, no wave amplitude. Therefore, values of zero for the amplitude should be ignored.

Overall, there are episodes of growth and decay of the waves for the Northern (Figure 6) and Southern (Figure 7) Hemispheres. Periods of growth seem to only last between 4 and 7 days. For example, in the Northern Hemisphere there is a large increase in the wave amplitude from September 29 to October 4, in which the wave amplitude increases from 104 m to 235 m (126% increase). Also, in the Southern Hemisphere the wave amplitude increases from 192.5 m to 317.5 m from October 9 to October 14 (65% increase).

There seems to be a correlation between the Northern and Southern Hemisphere amplitudes. The local maximums and minimums of the amplitudes appear to be one or two days apart (Figure 8) from one another. This observation does not occur for every time period of the domain, for example, between October 24 and October 29, but there seems to be a general trend.

c. Wave Number

Wave number also varies over a temporal scale. Wave number generally decreases over the domain in both the Northern (Figure 9) and Southern (Figure 10) Hemispheres. The range of
the wave number is approximately the same in both hemispheres, where the Northern Hemisphere varies from one to six waves and the Southern Hemisphere varies from zero to six waves. The average wave number in the Northern Hemisphere is 3.48 while it is only 3.06 in the Southern Hemisphere. Thus, on average the Northern Hemisphere has more waves. The wave number in the Northern Hemisphere is within +/- 1 of its average value from October 9th to October 25th. The rest of the period deviates by more than one increment from the average. For the Southern Hemisphere, the wave number is within +/- 1 of the average value from September 15 to September 22, September 25 to October 3, and October 29 to November 11. The periods when the wave number is +/- 1 of the average wave number do not agree with synoptic features. Synoptic features last around four to seven days. These periods extend for sixteen days in the Northern Hemisphere, and for seven to thirteen days in the Southern Hemisphere.

Figure 9

<table>
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</tr>
</tbody>
</table>

\[ y = -0.0202x + 828.03 \]

\[ R^2 = 0.13615 \]
The amplitude has a dependence on the wave number. In both hemispheres, as the wave number increases, the amplitude generally decreases (Figure 11 and Figure 12). This agrees with the hypothesis and makes sense with the Rossby Wave Theory. More short waves are generally present as the wave number increases, so the average amplitude of the waves will decrease. Short waves have smaller amplitudes than long waves because energy must be conserved. If there are more waves (more short waves), then each wave must carry less energy with it. Thus, the amplitude of the waves must decrease. The Northern Hemisphere data agrees with this correlation more than the Southern Hemisphere, because the coefficient of determination is 0.40745 while only 0.0072 in the Southern Hemisphere. The low coefficient of determination in the Southern Hemisphere may be due to the value of zero plotted for a wave number. This point also has a value of zero for the amplitude, so it is an outlier, which decreases the slope of the trend line to one that does not cover the general trend of the other data as well.

Figure 10
Figure 11

Amplitude vs. Wave Number NH

\[ y = -23.312x + 240.82 \]
\[ R^2 = 0.40745 \]

Figure 12

Amplitude vs. Wave Number SH

\[ y = -3.3737x + 200.36 \]
\[ R^2 = 0.0072 \]
\textit{d. Wave Motion}

The wave number also correlates with the wave motion. In the Northern Hemisphere as the wave number increases, the wave motion generally increases as well (Figure 13). This agrees with the hypothesis and makes sense with the Rossby Wave Theory. From equation (1), notice that as the wave number increases, the wave motion increases in the positive x-direction. Thus, short waves, which have a larger wave number than long waves, travel faster than long waves.

The opposite case appears in the Southern Hemisphere, where the wave motion decreases as the wave number increases (Figure 14). This does not agree with the hypothesis or the Rossby Wave Theory. As the wave number increases, the wave motion should also increase. This discrepancy may be explained by noting how the wave motion was calculated. Waves that retrograded, remained stationary, or decreased in amplitude (to the point of disappearance) were ignored. Thus, the wave motion data may not have been a true representation of what actually occurred over the domain. In addition, the Rossby Wave Theory is valid for barotropic flow, which means that a wave has the same density throughout its depth. This assumption may not be valid for the real atmosphere.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Wave_Motion_vs_Wave_Number_NH}
\caption{Wave Motion vs. Wave Number NH}
\end{figure}
Wave motion has a dependence on the 500 hPa zonal winds. Both the Northern (Figure 15) and Southern (Figure 16) Hemispheres show a slight negative correlation between these parameters. For positive values of 500 hPa zonal flow, as the 500 hPa wind increases, wave motion decreases. For negative values of 500 hPa zonal flow, as the 500 hPa wind increases (less negative), the wave motion decreases. The Southern Hemisphere follows this trend better than the Northern Hemisphere, because the coefficient of determination is 0.03063 compared to 0.00825 in the Northern Hemisphere. This does not agree with the hypothesis or the Rossby Wave Theory. Rossby Wave Theory states that waves propagate westward if no zonal flow is present. Only when the zonal flow reaches approximately 3 m/s does a wave begin to move eastward, depending on the wavelength. Thus, the theory says that as the westward zonal wind increases (more negative zonal wind), the wave should move faster toward the west. The data shows that as the westward zonal wind increases, the wave moves faster toward the east. The data also disagrees when the 500 hPa zonal wind is positive. According to Rossby Wave Theory, as the 500 hPa zonal flow moving eastward increases, the wave should propagate faster toward the east. For both hemispheres, as the 500 hPa zonal flow increases, the wave motion toward the east decreases. Although the linear fit is not strong, with coefficients of correlation of 0.00825 and 0.03063 for the Northern and Southern Hemispheres, respectively, the general trend over the domain disagrees with Rossby Wave Theory. Again, this discrepancy may be due to the way wave motion was calculated.

Figure 14
Figure 15

Wave Motion vs. 500 hPa Zonal Wind NH

\[ y = -0.0416x + 10.86 \]
\[ R^2 = 0.00825 \]

Figure 16

Wave Motion vs. 500 hPa Zonal Wind SH

\[ y = -0.0598x + 15.961 \]
\[ R^2 = 0.03063 \]
The average 500 hPa zonal wind is $5.06^\circ$/day in the Northern Hemisphere, while it is $27.21^\circ$/day in the Southern Hemisphere. The Rossby Wave Theory states that as the eastward zonal wind increases, the speed of the wave increases in the positive x-direction. The observed wave motion supports the theory. The average wave motion is lower in the Northern Hemisphere, $10.32^\circ$/day, than in the Southern Hemisphere, $14.01^\circ$/day, because the average zonal wind in the Southern Hemisphere is larger. Also, for the Southern Hemisphere the average 500 hPa zonal wind is greater than the average wave motion. This agrees with Rossby Wave Theory. A wave propagates westward if no zonal flow is present. When there is an eastward zonal flow, the wave will move eastward, but at a slower speed than the zonal wind. For the Northern Hemisphere the average 500 hPa zonal wind is less than the average wave motion. This does not agree with the Rossby Wave Theory. The average zonal wind should be larger than the average wave motion. This discrepancy is probably because the analysis procedure did not include waves that retrograd. Thus, there were no negative values for the wave motion. Negative values were included for some of the 500 hPa zonal winds, which decreased the overall average zonal wind. Thus, the magnitude of the average zonal wind is slightly less than the average wave motion.

The wave motion in the Northern Hemisphere, which is generally eastward, is smaller (on average) than the wave motion in the Southern Hemisphere, which is generally eastward as well. The wave motion in the Northern Hemisphere is only $10.32^\circ$/day, while it is $14.01^\circ$/day in the Southern Hemisphere. Thus, on average waves in the Southern Hemisphere travel faster than waves in the Northern Hemisphere. Given these averages, it would take a wave 34.88 days to travel around the earth in the Northern Hemisphere, while only 25.69 days in the Southern Hemisphere. Thus, weather systems will likely move faster through a region in the Southern Hemisphere.

The wave motion also has a dependence on the upper-level 150-300 hPa zonal wind ($U_{\text{upper}}$). Both the Northern (Figure 17) and Southern (Figure 18) Hemispheres show a slight negative correlation between these parameters. As the 150-300 hPa wind increases, wave motion decreases. This does not agree with the Rossby Wave Theory, as the trend over the domain is similar to the 500 hPa zonal flow. The theory states that as the westward zonal wind increases (more negative zonal wind), the wave should move faster toward the west. The data shows that as the westward zonal wind increases, the wave moves faster toward the east. The data also disagrees when the 150-300 hPa zonal wind is positive. According to the Rossby Wave Theory, as the 150-300 hPa zonal flow moving eastward increases, the wave should propagate faster toward the east. For both hemispheres, as the 150-300 hPa eastward zonal flow increases, the eastward wave motion decreases. Although the linear fit is not strong, with coefficients of correlation of 0.00496 and 0.00331 for the Northern and Southern Hemispheres, respectively, the general trend over the domain disagrees with the Rossby Wave Theory. Again, this discrepancy may be due to the way wave motion was calculated, or from assuming a simple barotropic atmosphere.
e. Thermal Wind

The thermal wind relationship states that whenever there is a horizontal temperature gradient, there will always be a change in the geostrophic wind with height. Higher winds will be
located at lower pressure levels. This study supports the thermal wind relationship for both the Northern (Figure 19) and Southern (Figure 20) Hemispheres. The wind speed for the 150-300 hPa level is almost always greater than for the 500 hPa level.
4. Conclusion

Overall, the results of this study vary in relation to the Rossby Wave Theory. There is agreement with the Rossby Wave Theory when comparing amplitude vs. time in both hemispheres, amplitude vs. wave number in both hemispheres, and wave motion vs. wave number in the Northern Hemisphere. The results disagree with Rossby Wave Theory when comparing wave motion vs. wave number in the Southern Hemisphere, wave motion vs. 500 hPa zonal winds for both hemispheres, and wave motion vs. 150-300 hPa zonal winds for both hemispheres. Also, two of the three hypotheses were supported by the observations. A larger wave number corresponds to a lower amplitude, and a larger wave number corresponds to a faster wave motion. The third hypothesis, that faster upper-level winds imply faster motion, was not supported by the observations. For many of these relationships the coefficient of determination was quite small (below 0.1), and outliers skewed the trends in some comparisons. Therefore, many of the correlations were weak and thus, cannot support definite conclusions. This study shows that while the Rossby Wave Theory is not perfect, it allows a meteorologist to forecast the weather with some degree of accuracy if the meteorologist is aware of its limitations.

References