

Problem Set 5

Due: 07 November 2016

1. Isothermal Sound Waves

The first time someone tried to derive the behavior of sound waves, it was assumed that they behave isothermally, rather than adiabatically. Thus, we would use $dT/dt = 0$, rather than $d\theta/dt = 0$. Making this change, we would arrive at

$$\begin{aligned}\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} &= 0 \\ \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} + p \frac{\partial u}{\partial x} &= 0\end{aligned}\tag{1a,b}$$

(a) Show that an acceptable basic state is $u = \bar{u}$, $p = \bar{p}$, $\rho = \bar{\rho}$, where all three fields are constants.

(b) Assume the isothermal sound waves are given by perturbations about this basic state: $u = \bar{u} + u'$, $p = \bar{p} + p'$, $\rho = \bar{\rho} + \rho'$. Substitute these forms for u , p , and ρ into (1a,b) and obtain the corresponding two linearized equations for the perturbation fields.

(c) If we eliminate variables in the two linearized equations to get a single equation in p' , we then have

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) \left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right) p' - \frac{\bar{p}}{\bar{\rho}} \frac{\partial^2 p'}{\partial x^2} = 0\tag{2}$$

As mentioned in class, the form $\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right)$ is a *differential operator*. The derivatives in it operate on whatever is to the right of it. When we see the form twice in a row, as in (2), it means we apply the rightmost $\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right)$ to p' , and then do it again with the next $\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x} \right)$. This is analogous to considering

$\frac{\partial^2 p'}{\partial x^2}$ as $\left(\frac{\partial}{\partial x} \right) \left(\frac{\partial p'}{\partial x} \right)$, where first we compute $\left(\frac{\partial p'}{\partial x} \right)$, and then take $\left(\frac{\partial}{\partial x} \right)$ of that result.

With that in mind, assume a wave solution, $p' = A \exp\{ik(x - ct)\}$, and derive the relationship for phase speed c in terms of \bar{u} , \bar{p} , and $\bar{\rho}$.