PROBLEMS 3 – CLIMATE MODELING

Due: 15 October 2015

1. Let's see how well various spatial distributions are represented by Legendre polynomials. We want to construct representations for functions that are symmetric about the equator (x=0), so we can assume they are can be represented by the form

$$f(x) \approx A_0P_0(x) + A_2P_2(x) + \ldots + A_nP_n(x) \qquad (1)$$

The relationship is exact if $n \rightarrow \infty$.

We can compute the coefficients $A_0, A_2, \ldots, A_n$ by recalling that

$$\int_0^1 P_n(x)P_{n'}(x)dx = \delta_{n,n'}\left(\frac{1}{2n+1}\right) = \begin{cases} 1/(2n+1) & n' = n \\ 0 & n' \neq n \end{cases} \qquad (2)$$

so then

$$\int_0^1 f(x)P_n(x)dx = \int_0^1 \{A_0 + A_2P_2(x) + \ldots + A_nP_n(x)\}P_n(x)dx = A_n\left(\frac{1}{2n+1}\right) \qquad (3)$$

yielding

$$A_n = (2n+1)\int_0^1 f(x)P_n(x)dx \qquad (4)$$

For this problem, recognize that the first 4 even Legendre polynomials are

$P_0(x) = 1$ $\quad$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$

$P_2(x) = (3x^2 - 1)/2$ $\quad$ $P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$

For each case below, make a plot of the temperatures given by f(x) and then, on the same graph, a plot of

$$F(x) = A_0P_0(x) + A_2P_2(x) + \ldots + A_nP_n(x)$$

using the A_n derived from the function. You should also give me to the values of the A_n needed for the plot.

(a) $f_1(x) = 288\ K$ (i.e., the average temperature of the earth).

Compute coefficients A_n for n = 0, 2, 4, 6. You can do this one without resorting to computing equation (4) if you think carefully about equation (2) and how f_1 varies with x.

(b) $f_2(x) = 302 - 42x^2$

Compute coefficients for n = 0, 2, 4 to produce F(x)

(c) $f_3(x) = \begin{cases} 288 & 0 \le x \le 0.5 \\ 268 & x > 0.5 \end{cases}$

Compute coefficients for n = 0, 2, 4, 6. Plot 3 curves: $f_3(x)$, F(x) using just n=0 and 2, and then F(x) using n = 0, 2, 4 and 6. Does the second F always give a closer match to f_3 than the first?

(d) Why do we only need to use the even polynomials for the functions examined here? That is, why aren't we also using $P_n(x)$ for n = 1, 3, 5, …?