PROBLEMS 2 – PHYSICS OF CLIMATE

Due: 24 October 2008

You are welcome (in fact encouraged) to consult with each other (as well as me) if you have difficulty working on these, but you must each hand in your own answer set. Please turn in your work via email to gutowski@iastate.edu. To help me spot it in my Inbox, like before, have the subject line read:

Subject: Physics of Climate, Problem Set 2

1. Assume there is no dynamic feedback in the 1-D, latitudinal energy balance model that uses two Legendre modes, P_0 and P_2. Instead, fix the dynamic flux at the magnitude and distribution it has for the two-mode model's simulation of the current climate (i.e., in any terms involving dynamic heat transport, specify the D and T_2 (or I_2) as given by the solution for the current climate). Note that T_2 (or I_2) is specified *only* in terms involving D, otherwise it is still free to vary. You might call the T_2 (or I_2) multiplying D something distinctive, like T_{2C} (or I_{2C}) to refer to the value fixed at the current climate's value (thereby fixing the dynamic flux, too). Otherwise, retain T_2 (or I_2) as a variable wherever else it appears in the model.

Given this restriction, use slides 10 and 11 of lecture 12 as a guide for obtaining the solution. Using North's parameters for other quantities like outgoing longwave radiation, calculate the resulting curves $x_i(Q_o)$ and $x_i(T_o)$ for $0.6 \le x_i \le 1.0$. Do the same for the case examined in the online lectures, where there is dynamic feedback. (The table below may be useful.)

What can you conclude about the importance of dynamic feedback, especially for climate stability?

TABLE A2. Values of functions required for hand computations of $Q(x_s)$, T_0, T_2, T_4 in all models retaining up to three modes. Albedo parameters used are $a_0=0.697$, $a_2=-0.0779$, $b_0=0.380$.

x_s	H_0	H_2	H_2P_2	H_4	H_4P_4
0.60	0.6174	−0.5825	−0.0233	0.1384	−0.0565
0.65	0.6323	−0.5761	−0.0771	0.0821	−0.0352
0.70	0.6461	−0.5635	−0.1324	0.0295	−0.0121
0.75	0.6587	−0.5452	−0.1874	−0.0145	−0.0051
0.80	0.6703	−0.5222	−0.2402	−0.0344	0.0106
0.85	0.6806	−0.4952	−0.2891	−0.0593	0.0030
0.90	0.6898	−0.4656	−0.3329	−0.0536	−0.0112
0.95	0.6977	−0.4345	−0.3709	−0.0272	−0.0151
1.00	0.7045	−0.4031	−0.4031	0.0193	+0.0193

Note: The albedo parameterization is the same as presented in the lectures, but here

$$1-\alpha = \begin{cases} a_0 + a_2 P_2(x) & x < x_i \\ b_0 & x > x_i \end{cases}$$

2. Let's see how well various spatial distributions are represented by Legendre polynomials. We want to construct representations for functions that are symmetric about the equator (x=0), so we can assume they are can be represented by the form

$$f(x) \approx A_0P_0(x) + A_2P_2(x) + \ldots + A_nP_n(x) \qquad (1)$$

The relationship is exact if $n \to \infty$.

We can compute the coefficients A_o, A_2, …, A_n be recalling that

$$\int_0^1 P_n(x)P_{n'}(x)dx = \delta_{n,n'}\left(\frac{1}{2n+1}\right) = \begin{cases} 1/(2n+1) & n' = n \\ 0 & n' \neq n \end{cases} \qquad (2)$$

so then

$$\int_0^1 f(x)P_n(x)dx = \int_0^1 \{A_0 + A_2P_2(x) + \ldots + A_nP_n(x)\}P_n(x)dx = A_n\left(\frac{1}{2n+1}\right) \qquad (3)$$

yielding

$$A_n = (2n+1)\int_0^1 f(x)P_n(x)dx \qquad (4)$$

For this problem, recognize that the first 4 even Legendre polynomials are

$P_0(x) = 1$

$P_2(x) = (3x^2 - 1)/2$

$P_4(x) = (35x^4 - 30x^2 + 3)/8$

$P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$

For each case below, make a plot of the temperatures given by f(x) and then, on the same graph, a plot of

$$F(x) = A_0P_0(x) + A_2P_2(x) + \ldots + A_nP_n(x)$$

using the A_n derived from the function. You should also give me to the values of the A_n needed for the plot.

(a) $f_1(x) = 288\ K$ (i.e., the average temperature of the earth).

Compute coefficients A_n for n = 0, 2, 4, 6. You can do this one without resorting to computing equation (4) if you think carefully about equation (2) and how f_1 varies with x.

(b) $f_2(x) = 302 - 42x^2$

Compute coefficients for n = 0, 2, 4 to produce F(x)

(c) $f_3(x) = \begin{cases} 288 & x \le 0.5 \\ 268 & x > 0.5 \end{cases}$

Compute coefficients for n = 0, 2, 4, 6. Plot 3 curves: $f_3(x)$, F(x) using just n=0 and 2, and then F(x) using n = 0, 2, 4 and 6. Does the second F always give a closer match to f_3 than the first?

(d) Why do we only need to use the even polynomials for the functions examined here? That is, why aren't we also using $P_n(x)$ for n = 1, 3, 5, …?