Exam – Physics of Climate

Time allowed: 120 minutes
You are allowed to use all online class materials, as well as graded problem sets and computer (EdGCM) labs.

1. [50 points] You are the science officer on the starship *Spes Bona*, orbiting a newly discovered planet, *Ciudad del Cabo.* You are using your radiation sensors to determine properties of the planet's climate. The atmosphere of *Ciudad del Cabo* is cloud-free, controlled by radiative fluxes (no dynamic heat transport) and overlies a surface consisting only of land (no oceans).

All parts of this problem except part (e) can be solved using Lecture 1.
(Side note: *Spes Bona* = Good Hope [Latin], the motto of the University of Cape Town and, of course, reference to the Cape of Good Hope and its historical significance to European explorers. *Ciudad del Cabo* = City of the Cape [Spanish], i.e., Cape Town.)

(a) Your sensors measure the shortwave radiation flux S_V (the "stellar constant") coming from the star of *Ciudad del Cabo.* It has a value of $S_V = 1200$ W-m^{-2}. While orbiting the planet, they also measure its upward longwave flux to space. It has a global average value $\bar{I}_V = 280\,\mathrm{W-m^{-2}}$. What is the planetary albedo, $\bar{\alpha}$?

Use

$$\bar{I} = \tfrac{1}{4}(1-\bar{\alpha})S$$

Then

$$\bar{\alpha} = 1 - 4\bar{I}/S$$

So albedo = 6.7%.

(b) What is $\bar{T}_{RAD}$ the radiative temperature of the planet?

T_{RAD} is the temperature the planet would have if it radiated like a black body with I = 280 W-m$^{=2}$. Use $\bar{T}_{RAD} = \left(\bar{I}/\sigma\right)^{1/4}$ = (280 W-m^{-2}/5.67$\cdot$10^{-8} deg-^{4}-W-m^{-2}) = 265 K.

(c) From probes sent to the surface, you determine that the global-average surface temperature is $\bar{T}_s = 400\,\mathrm{K}$. What is the effective emissivity of *Ciudad del Cabo*? [Be careful with your units!]

Now use $\bar{I} = \varepsilon\sigma\left(\bar{T}_s\right)^4$ to link outgoing radiation with surface temperature. That is, assume the planet radiates like an equivalent black body at the temperature of the surface. Then, $\varepsilon = \bar{I}/\sigma\left(\bar{T}_s\right)^4$ = 280 W-m^{-2} / { 5.67$\cdot$10^{-8} deg-^{4}-W-m^{-2} x (400 deg)^{4} } = 0.19.

(d) Because the surface temperature is above the boiling point of water, the habitation team proposes sprinkling the surface with a highly reflection medium, thereby increasing the planetary albedo. Assuming the effective emissivity does not change, what must be the new planetary albedo $\bar{\alpha}$ to have a surface temperature of 300 K?

Incoming radiation $= (1-\bar{\alpha})\frac{S}{4}$ Outgoing radiation = $\varepsilon\sigma\left(\bar{T}_s\right)^4$

We know all parameters except albedo, so using the balance requirement:

outgoing radiation = incoming radiation

We get $\bar{\alpha} = 1 - \varepsilon\sigma\bar{T}_s^4\frac{4}{S}$ = 70.9%

(e) How would you set up a set of EdGCM simulations to test your result in part (d)?

We would first have to set up a planet that had an albedo of 6.7% so that incoming radiation due to a stellar constant of 1200 W-m^{-2}, the outgoing radiation would be 280 W-m^{-2}. Then we would have to alter the atmosphere so that surface temperature originally was 400 K, perhaps by changing the mass of the atmosphere (surface pressure) until this temperature occurred, or perhaps by altering the composition of greenhouse gases. We would have to simulate several years to get to a statistical equilibrium, and then several years more to be sure we had an average surface temperature of 400 K.

Once we had that, we could change the surface albedo to 70.9% and run the model to its new equilibrium, checking that the temperature did drop eventually to an average value of 300 K.

2. [30 points] Consider the zero-dimensional global energy balance model we studied.

All parts of this problem can be solved using Lecture 5, though of course knowing material from the previous and later lectures helps.

(a) For the stable solution that approximates our present climate, how does global average temperature change with solar constant using the Modified Sellers parameterization? Present this as a table of S and $\bar{T}_s$ values or a simple plot of $\bar{T}_s$ vs. S. [Be careful to determine the lowest allowable $\bar{T}_s$ and hence lowest allowable S for this stable branch of the possible solutions.]

We have

$$\boxed{A + B\bar{T}_s = \frac{S}{4}\left\{1-\bar{\alpha}\left(\bar{T}_s\right)\right\}}$$

with

$$\boxed{\bar{\alpha} = \begin{cases} \bar{b} & \bar{T}_s > 10^\circ\mathrm{C} \\ \bar{b} - \bar{d}\left(\dfrac{\bar{T}_s - 10}{39}\right) & -29^\circ\mathrm{C} < \bar{T}_s < 10^\circ\mathrm{C} \\ \bar{b} + \bar{d} & \bar{T}_s < -29^\circ\mathrm{C} \end{cases}}$$

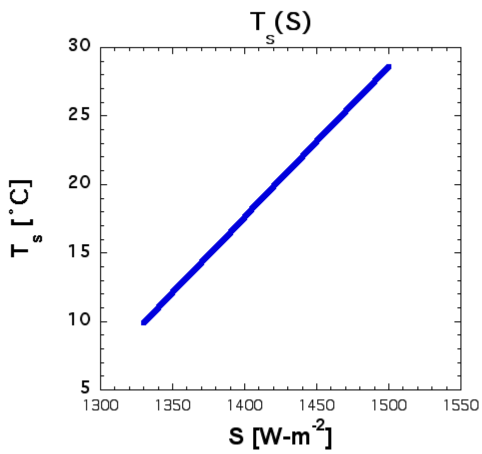
where the parameters with overbars b = 0.30 and d = 0.20. Also,

$$\boxed{\bar{I}\left(\mathrm{W/m^2}\right) = 217 + 1.59\bar{T}_s\left(^\circ\mathrm{C}\right)}$$

For the stable solution of this model that approximates our present climate, we are in the regime where average T_s > 10˚C, so albedo is then a simple constant, b = 0.30. We can then easily solve for $T_s(S)$:

$$\boxed{\bar{T}_s = \left\{\frac{S}{4}\left\{1-\bar{b}\right\} - A\right\} \Big/ B}$$

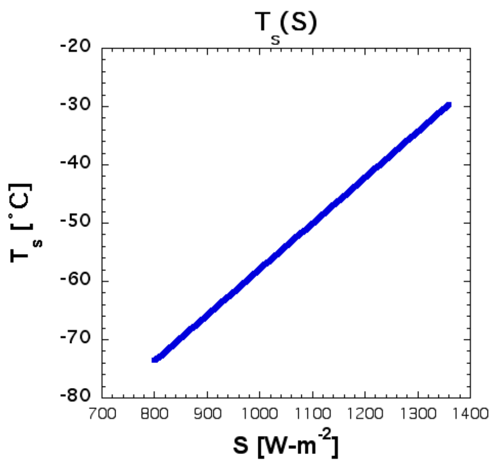
The lowest allowable value of T_s is the lower limit for this part of the albedo range with fixed albedo, of T_s = 10˚C. (If T_s falls below that, then the albedo has to have values given by the other temperature ranges in the modified Sellers parameterization.) Knowing this, we can then compute average surface temperature as a function of S:



Note that the figure shows $T_s(S)$ and not $S(T_s)$. This is not a trivial distinction. The latter implies that changes in T_s cause S to vary, which is not physical. The former implies that changes in S cause T_s to vary, which does make physical sense. This is why this figure has T_s, the dependent variable, on the y-axis and S, the independent variable, on the x axis.

(b) Suppose S drops below the lower limit on it in part (a), what then becomes the stable climate for the earth?

The lower limit for S in part (a) is about 1330 W-m^{-2}. If S should decrease below this limit, then T_s decreases and puts us in the range where albedo varies with temperature. However, we have seen that this is an unstable regime, so temperature will continue to drop, and albedo will increase until the planet falls into the other stable regime, where albedo is now = b+d = 0.5. This gives



Note that now, for S ≈ 1330 W-m^{-2}, the surface temperature is much colder: the climate jumps down to a new, colder temperature. There is, in fact, a range of S (~1330 – 1365 W-m^{-2}) where there are two possible solutions: a very cold planet, with more snow/ice cover, and a warmer planet, approximately like ours. Which one the climate will end at depends on whether it is perturbed toward the ice-free (low albedo) or ice-covered (high albedo) state.

3. [20 points] Let's go back to Mars and planetary engineering. Our final estimate of the longwave optical depth needed to give a surface temperature of 0˚C was $\tau \approx 8$. The longwave optical depth for the earth's atmosphere is about $\tau \approx 4$, and this is due primarily to water vapor. Let's suppose we need $\tau \approx 8$ for Mars, and that we expect water vapor to give this. How much mass of water vapor will we need and is that water present on Mars?

We can evaluate these questions with a few computations you are about to do:

(i) estimate mass of earth's atmosphere

(ii) estimate mass of water in the Martian North Pole ice cap

(a) The average density of water at the earth's surface is approximately $\rho_{wo} \approx 1.2 \cdot 10^{-3}$ kg/m^3. Water vapor varies with height approximately as $\rho_w(z) \approx \rho_{wo} \exp\{-z/h_w\}$, where $h_w \approx 2$ km. Thus, the average column mass of water vapor (units: mass/area) is

$$PW = \int_0^\infty \rho_w(z)dz$$

What is the mass of water vapor for the entire area of the earth? Presumably we would need to double this to raise the longwave optical depth for earth from $\tau \approx 4$ to $\tau \approx 8$ (as an approximation).

We get the mass of water vapor for the entire planet by integrating the PW equation, which gives the mass per unit area, and then multiplying by the surface area of the earth. Thus,

$PW = \rho_{wo}h_w$,

using the integration formula below. The surface area of the earth is $4\pi(R_{earth})^2$. Thus total mass = PW x area = $(\rho_{wo}h_w)\{4\pi(R_{earth})^2\} = 1.2 \cdot 10^{15}$ kg.

(b) The Martian North Pole has the planet's largest ice cap. One estimate from a Martian orbiter (http://ltpwww.gsfc.nasa.gov/tharsis/agu_f98.html Note: site no longer exists) is that the ice has a volume of $1.2 \cdot 10^6$ km^3. The density of ice is ~ $0.92 \cdot 10^3$ kg/m^3.

What is the mass of Martian North Pole ice?

This is simply $M_{Mars} = 1.2 \cdot 10^6$ km^3 x (1000 m/km)^3 x $0.92 \cdot 10^3$ kg/m^3 = $1.10 \cdot 10^{18}$ kg.

(c) If the Martian North Pole ice evaporated, how would it compare to the volume of water estimated necessary to raise the earth's longwave optical depth to $\tau \approx 8$?

This is more than enough, according to these computations.

(d) Final point: the surface area of Mars is about ¼ the surface area of Earth. What might this mean for the volume of water vapor we need to raise the longwave optical depth to $\tau \approx 8$?

If we assume the scale height for water vapor in our "engineered" Martian atmosphere is about the same as for earth (a reasonable assumption given that the temperatures would be approximately the same in the lowest km or 2), then for asurface area only ¼ that of the earth, we need only ¼ of the mass, about $3 \cdot 10^{14}$ kg, which further make the ice cap on Mars more than sufficient as a supplier of water vapor.

Possibly useful numbers and equations

$\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W-m^{-2}-K^{-4}}$ $\quad \pi = 3.141592654 \quad e^{-1} = 0.36787945$

$R_{earth} = 6.37 \cdot 10^6$ m

$$\int_a^b \exp\{-z/H\}dz = H\{\exp(-a/H) - \exp(-b/H)\}$$