Student Compiled Potential Exam II Questions (Some will very likely be on the exam!)

Thermodynamics of the dry atmosphere (2.7 Holton)
Nobody submitted questions in this category, but we'll go over relevant material in the review.

Basic Equations in Isobaric Coordinates (3.1 Holton)

- What are the advantages and disadvantages of converting the basic equations to isobaric coordinates?

answer: In the horizontal momentum equation and the continuity equation references to density are removed making things simpler. In addition, the continuity equation in isobaric coordinates no longer has a time derivative making things even simpler. The only disadvantage is in the conversion of the thermodynamic equation to isobaric coordinates. A reference to density is added which appears in the stability term, Sp. Thus, the stability parameter has a strong dependence on height (since density decreases exponentially with height) which is a minor disadvantage for isobaric coordinates.

- Starting with the horizontal momentum equation in vectorial form,

$$\frac{d\vec{V}}{dt} + f\hat{k}\times\vec{V} = -\frac{1}{\rho}\nabla p\,,$$

derive the geostrophic balance equation in isobaric coordinates.

answer: $\frac{1}{\rho}\nabla p = \nabla_p \Phi$ and $\frac{d\vec{V}}{dt} = 0$ (no acceleration when motion is geostrophic) giving:

$f\hat{k}\times\vec{V} = -\nabla_p\Phi$ take $\hat{k}\times$ of both sides obtaining…

$f\left[\hat{k}\times(\hat{k}\times\vec{V})\right] = -\hat{k}\times\nabla_p\Phi$ apply vector identity $A\times(B\times C) = B(A\bullet C) - C(A\bullet B)$ to LHS…

$f[\hat{k}(\hat{k}\bullet\vec{V}) - \vec{V}(\hat{k}\bullet\hat{k})] = -\hat{k}\times\nabla_p\Phi$ $\hat{k}\bullet\vec{V} = 0$ and $\hat{k}\bullet\hat{k} = 1$

$-f\vec{V} = -\hat{k}\times\nabla_p\Phi$

Balanced Flow (3.2 Holton)

- Explain the difference between geostrophic, inertial, and cyclostrophic flow using the force balances derived in the natural coordinate system.

answer: Geostrophic flow is straight line flow in which the height contours have no curvature; thus, the centrifugal acceleration term is zero leaving a balance between the coriolis force and the pressure gradient force. Inertial flow occurs when there is uniform pressure, thus, the pressure gradient force goes to zero leaving a balance between the coriolis force and the centrifugal force. Cyclostrophic flow occurs when motions are small enough scale to neglect the coriolis force leaving a balance between the centrifugal acceleration and the pressure gradient force. See below…

Geostrophic Flow

$$\underbrace{\cancel{\frac{V^2}{R}}}_{0} + fV = -\frac{\partial\Phi}{\partial n}$$

Inertial Flow

$$\frac{V^2}{R} + fV = \underbrace{-\cancel{\frac{\partial\Phi}{\partial n}}}_{0}$$

Cyclostrophic Flow

$$\frac{V^2}{R} + \underbrace{\cancel{fV}}_{0} = -\frac{\partial\Phi}{\partial n}$$

- How can we use the Rossby number, V/fR, to gain information on the validity of the cyclostrophic balance approximation? How is the application of the Rossby number to the cyclostrophic balance approximation different from its application to the geostrophic wind approximation?

answer: If the Rossby number is large, the cyclostrophic balance approximation is valid. This is different from the geostrophic wind approximation in which small Rossby numbers indicate that the geostrophic wind is accurate.

- Give the sign of the pressure gradient term, $\partial\Phi/\partial n$, and radius of curvature term, R, and root for all physical solutions of the gradient wind approximation,

$$V = -\frac{fR}{2} \pm \left(\frac{f^2 R^2}{2} - R\frac{\partial\Phi}{\partial n} \right)^{1/2}$$

Also, give the name of the phenomenon that each solution describes.

answer:

regular low; R>0, $\partial\Phi/\partial n < 0$, *positive root.*

anomalous low; R<0, $\partial\Phi/\partial n > 0$, *positive root.*

regular high; R>0, $\partial\Phi/\partial n < 0$, *negative root.*

anomalous high; R>0, $\partial\Phi/\partial n < 0$, *positive root.*

- What does the following equation,

$$\frac{d\vec{V}}{dt} = \hat{t}\frac{dV}{dt} + \hat{n}\frac{V^2}{R},$$

which we derived in terms of the natural coordinate system, tell us?

answer: The acceleration following the motion equals the sum of the rate of change in speed and the centripetal acceleration due to the change in trajectory.

Trajectories and Streamlines (3.3 Holton)

- How are trajectories and streamlines different?

answer: Trajectories trace the motion of individual fluid particles, while streamlines are parallel to the instantaneous wind velocity.

- Why does using the geopotential height gradient to calculate the gradient wind typically lead to errors?

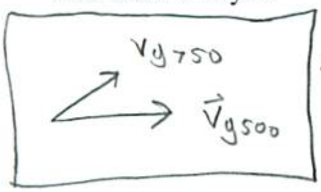
answer: The radius of curvature term, R, in the gradient wind equation is for the radius of curvature of a parcel trajectory. When we use the geopotential height gradient to estimate R we are using streamlines, or the instantaneous wind velocities, which can be very different from the parcel trajectories and can cause errors in the gradient wind approximation.

Thermal Wind (3.4 Holton)

- What is the thermal wind?

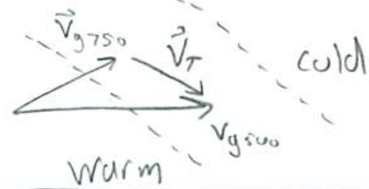
answer: The thermal wind is the difference between the geostrophic winds at two levels. The thermal wind runs parallel to contours of the layer mean temperature between the two levels thermal wind is being calculated for.

- Using the diagram below of geostrophic wind vectors at 500mb and 750mb, show how you can use the thermal wind equation to find out what type of temperature advection is likely occurring in the 750-500mb layer.



answer→

$$\vec{V}_T = \vec{V}_{g\,500} - \vec{V}_{g\,750} = \frac{R}{f}\underbrace{\hat{K}\times\nabla\langle T\rangle}\ln{}^{p_0}/_{p_1}$$



T contours are parallel to $\vec{V}_T$ w/ warm air on right.

answer: From the diagram constructed based on the thermal wind equation, it is obvious WAA is occurring

- Using the component equations for the geostrophic wind and the hydrostatic balance approximation, derive the thermal wind equation.

answer:

component eqs. for geostrophic wind → $\left[v_g = \frac{1}{f}\frac{\partial\Phi}{\partial x} \quad u_g = -\frac{1}{f}\frac{\partial\Phi}{\partial y} \right]$

hydrostatic balance equation → $\left[\frac{\partial p}{\partial z} = -\rho g\right]$ convert to different form...

$\frac{\partial p}{\partial z} = -\rho g$ → divide by g ...

[use $g\,dz = d\Phi$]

$\frac{dp}{d\Phi} = -\rho$

[take the inverse remember $\frac{1}{\rho} = \alpha$]

$\frac{d\Phi}{dp} = -\alpha$ → substitute $\alpha = \frac{RT}{p}$ from ideal gas law

$\left[\frac{\partial\Phi}{\partial p} = -\frac{RT}{p}\right]$ → we'll use later

Differentiate expressions for v_g and u_g w.r.t. pressure. I'll only do for v_g in this example ---

$\frac{\partial v_g}{\partial p} = \frac{\partial}{\partial p}\left(\frac{1}{f}\frac{\partial\Phi}{\partial x}\right)$ → change order of differentials → f constant

$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\frac{\partial\Phi}{\partial p}$ → make substitution from hydrostatic equation.

$\frac{\partial v_g}{\partial p} = \frac{1}{f}\frac{\partial}{\partial x}\left[-\frac{RT}{p}\right]$ → multiply by p

$\boxed{\frac{\partial v_g}{\partial \ln p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p}$ → a similar procedure is applied to find u_g obtaining

$\boxed{\frac{\partial u_g}{\partial \ln p} = \frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p}$

These two equations can be combined to get the thermal wind equation in vectorial form.

$$\boxed{\frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R}{f}\hat{K}\times\nabla_p T}$$

Baroclinic and Barotropic Atmospheres (3.4.1 Holton)

- Explain the difference between barotropic and baroclinic atmospheres? Is the real atmosphere typically barotropic or baroclinic?

answer: In a barotropic atmosphere density is only a function of pressure, $\rho = \rho(p)$. In a baroclinic atmosphere density is a function of both pressure and temperature, $\rho = \rho(p,T)$. The real atmosphere is typically baroclinic.

- Use the thermal wind equation,

$$\frac{\partial \vec{V}_g}{d \ln p} = -\frac{R}{f}\hat{k} \times \nabla_p T$$

along with what you know about ideal gases to explain the constraint that barotropy has on rotating fluids.

answer: For an ideal gas, isobaric surfaces will also be isothermal if the atmosphere is barotropic. Thus, the temperature gradient on an isobaric surface is zero so that $\frac{\partial \vec{V}_g}{d \ln p} = 0$. This implies that the geostrophic wind has no vertical shear in a barotropic atmosphere. So, in a rotating fluid, the rotation will be the same at all levels in the atmosphere.

Vertical Motion (3.5 Holton)

- What are the flaws in using the kinematic method,

$$\omega(p) = \omega(p_s) + (p_s - p)\left(\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right)_p,$$

and adiabatic method,

$$\omega = S_p^{-1}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right),$$

for estimating vertical motion.

answer: For the kinematic method, small errors in the horizontal wind components will lead to large errors in the estimation of divergence which will lead to large errors in the vertical motion estimate. For the adiabatic method, the local rate of change in temperature is needed which is difficult to accurately estimate over a wide area unless observations are taken very frequently. Also, this method is inaccurate when strong diabatic heating is present.

- Derive the kinematic method for vertical motion starting with the continuity equation.

answer:

Continuity equation $\rightarrow \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$ rearrange ...

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p$$

integrate from p_s to p

$$\int_{p_s}^{p} \frac{\partial \omega}{\partial p}\, dp = -\int_{p_s}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp$$

Let $u = \langle u \rangle \equiv$ average u from p_s to p

" $v = \langle v \rangle \equiv$ " v " " " "

$$\omega(p) - \omega(p_s) = -(p - p_s)\left[\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right]_p$$

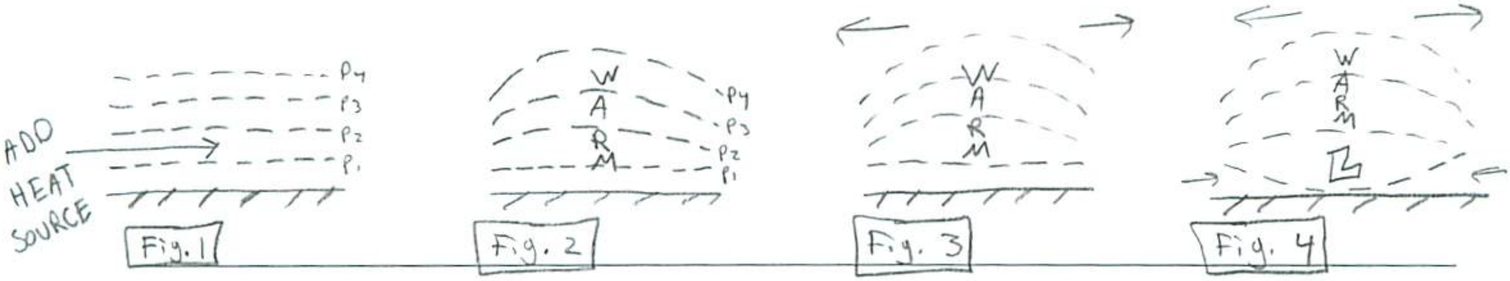
rearrange things a little

$$\boxed{\omega(p) = \omega(p_s) + (p_s - p)\left[\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right]_p}$$

Surface Pressure Tendency (3.6 Holton)

- Use the pressure tendency equation (Eq. 1) to explain/show how a "heat low" forms. Use pictures. Start by considering an initial state with pressure surfaces oriented horizontally and add a heat source to the mid-troposphere of this initial state (Fig. 1).

answer: The hypsometric equation tells us that heights above the warm anomaly must rise (Fig. 2). The raised pressure surfaces create a horizontal pressure gradient that drives a divergent upper-level wind (Fig. 3). The surface pressure tendency equation tells us that the upper-level divergence will lead to a drop in surface pressure (Fig. 4) and the formation of a "heat low".



Circulation Theorem (4.1 Holton)

- What are the fundamental differences between the two ways to measure rotation in a fluid: circulation and vorticity.

answer: Circulation is a scalar integral calculated over a finite area and is referred to as a macroscopic quantity. Vorticity is 3-d vector calculated at any point in a fluid and is a microscopic quantity

- What is the advantage to using circulation as a measure of rotation as opposed to angular momentum or angular velocity?

answer: Circulation can be computed without reference to an axis of rotation.