Good Exam I questions compiled by Mteor 443 students (Some of these will very likely be on the exam!)

Fundamental Forces and Apparent Forces

- What are the three fundamental forces, the two apparent forces, and where do the apparent forces come from?

answer: Fundamental forces include the pressure gradient force, gravitational force, and frictional force. Apparent forces include the centrifugal force and the coriolis force which originate because of the Earth's rotation.

- Describe the nature of each fundamental and apparent force.

answer:
*Pressure gradient force: Force proportional to the gradient in pressure. It is directed from high to low pressure. ***You should also be able to derive this force so that it is in terms of geopotential height. This is left as an exercise for you to complete.****
Gravitational force: proportional to mass and inversely proportional to the distance from the mass
Frictional force: Force that causes resistance of fluid to flow. We showed that frictional force was related to vertical wind shear.

- What is the difference between the centripetal and centrifugal acceleration?

answer:Centrifugal force has the same magnitude, but opposite sign as the centripetal force. Also, the centripetal force is a real force while the centrifugal is an apparent force.

- What would be the maximum value of the Coriolis parameter for a body the same size as Earth but rotating twice as fast?

answer: The coriolis parameter is $f = 2\Omega \sin\phi$. The maximum value occurs when $\sin\phi = 1$ ($\phi = 90^\circ$). Thus, $\Omega = 2\Omega_{Earth}$.

$$f_{\max} = 4\Omega_{Earth} = 4(7.292x10^{-5}\,rad/s) = 2.92x10^{-4}\,rad/s$$

- Explain the difference between the gravity force and the gravitational force.

answer: The gravitational force is a real force directed towards the exact center of the Earth. The gravity force is an apparent force because it takes into account the centrifugal force; because of the centrifugal force the gravity force is directed slightly away from the exact center of Earth. However, the gravity force is always perpendicular to the local geopotential surfaces, because the Earth has taken on the shape of an oblate spheroid. The gravitation force is not perpendicular to the local geopotential surfaces and actually has a component directed toward the poles.

- What is a geocentric reference frame?

answer: The geocentric reference frame is a frame of reference at rest with respect to the rotating Earth. It is referred to as a non-inertial reference frame because Newton's 1st law does not apply. Motion is accelerated in this frame, not uniform. The motion is accelerated because of the Earth's rotation.

- Explain why the coriolis force is small for some types of motion and not for others.

answer: For motions that have small time scales compared to the Earth's rotation, the coriolis force is negligible (cumulus clouds and tornadoes are examples of phenomenon with small time scales compared to the Earth's rotation).

- We can show that the gravitational force, g^*, can be expressed as:

$g^* = \frac{g_0^*}{(1+\frac{z}{a})^2}$ where g_0^* is the gravitational force at mean sea level, z is the distance from mean sea level and a is the radius of the Earth. Explain why for meteorological applications we can treat the gravitational force as a constant.

answer: For meteorological applications, z<<a so z/a is small. As a result $g^* = g_0^*$ *, so we can treat the gravitational force as a constant.*

- From the point of view of an observer in inertial space, a ball tied to a string is being twirled through the air by your hand. Does the ball have a constant speed and velocity?

answer: The speed of the ball is constant, but the direction of the ball is always changing, making the velocity not constant.

Hydrostatic equation and derivation of hypsometric equation

- Show that thickness is proportional to layer mean temperature (Hint: Start with hydrostatic balance equation).

answer: $\frac{dp}{dz} = -\rho g$ *(hydrostatic balance equation)*

Rearrange and substitute $\rho = \frac{p}{RT}$ *and* $d\Phi = gdz$ *to obtain:*

$dp = -\frac{p}{RT} d\Phi$ *Rearrange again and convert to differential form (i.e.* $\frac{dp}{p} = d\ln p$ *),*

$d\Phi = -RTd\ln p$ *Integrate both sides and obtain,*

$\Phi(z_2) - \Phi(z_1) = g_0(Z_2 - Z_1) = R\int_{p_2}^{p_1} Td\ln p$ *Note, we have defined:*

$Z \equiv \Phi(z)/g$ *as geopotential height. Defining thickness as* $Z_T \equiv Z_2 - Z_1$*, we obtain:*

$Z_T = \frac{R}{g_0}\int_{p_2}^{p_1} Td\ln p$ *If we define* $\langle T \rangle$ *as a layer mean temperature we obtain:*

$Z_T = \frac{R\langle T \rangle}{g_0} \ln p_1 / p_2$ *which shows that thickness is proportional to layer mean temperature.*

Total differentiation

- The temperature 30 km north of a point is 2°C cooler than at the point. If the wind is blowing horizontally at 10m/s from 300° and the air is warming by radiation at $0.5^\circ C/hr$, what is the local temperature change at the point?

answer: use the definition of the total derivative $\frac{\partial T}{\partial t} = \frac{dT}{dt} - (u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z})$

The terms $u\frac{\partial T}{\partial x}$ *and* $w\frac{\partial T}{\partial z}$ *go to zero because there is no East/West T gradient and only horizontal motions, respectively.*

Thus, $\frac{\partial T}{\partial t} = \frac{dT}{dt} - v\frac{\partial T}{\partial y}$

$\frac{dT}{dt} = 0.5^\circ C/hr$ *and* $\frac{\partial T}{\partial y} = -2^\circ C/30km$. *We must find the v-component of the wind:*

$v = (-10m/s)\cos 60^\circ = (-5m/s)(1km/1000m)(3600s/1hr) = -18km/hr$

$\frac{\partial T}{\partial t} = 0.5^\circ C/hr - (-18km/hr)(-2^\circ C/30km) = -0.7^\circ C/hr$

Total differentiation of a vector in a rotating coordinate system and application to the momentum equation in rotating coordinates.

- Using the expression relating the total derivative of a vector, $\vec{A}$, in an inertial frame, to the total derivative of the same vector in a rotating frame, which is,

$\frac{d_a\vec{A}}{dt} = \frac{d\vec{A}}{dt} + \vec{\Omega}\times\vec{A}$ (where $\frac{d_a}{dt}$ and $\frac{d}{dt}$ denote the total derivatives in the inertial and rotating frame, respectively) derive the vectorial form of the momentum equation in rotating coordinates.

answer: Apply the previous expression to a position vector, $\vec{r}$, on the rotating Earth obtaining:

$\frac{d_a\vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{\Omega}\times\vec{r}$ *since,* $\frac{d_a\vec{r}}{dt} = \vec{V}_a$ *and* $\frac{d\vec{r}}{dt} = \vec{V}$,

$\vec{V}_a = \vec{V} + \vec{\Omega}\times\vec{r}$. *Apply the expression above relating the total derivative in an inertial and rotating reference frame to the velocity vector in the inertial frame, $\vec{V}_a$, and obtain:*

$\frac{d_a\vec{V}_a}{dt} = \frac{d\vec{V}_a}{dt} + \vec{\Omega}\times\vec{V}_a$. *Substitute* $\vec{V}_a = \vec{V} + \vec{\Omega}\times\vec{r}$ *on the right hand side and obtain:*

$$\frac{d_a\vec{V}_a}{dt} = \frac{d}{dt}(\vec{V} + \vec{\Omega}\times\vec{r}) + \vec{\Omega}\times(\vec{V} + \vec{\Omega}\times\vec{r}) \qquad \textit{Expand....}$$

$$\frac{d_a\vec{V}_a}{dt} = \frac{d\vec{V}}{dt} + \vec{\Omega}\times\frac{d\vec{r}}{dt} + \vec{\Omega}\times\vec{V} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}) \qquad \textit{Simplify...}$$

$$\frac{d_a\vec{V}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega}\times\vec{V} - \Omega^2\vec{R}$$

$$\frac{d_a\vec{V}_a}{dt} = \sum F = -\frac{1}{\rho}\nabla p + g^* + F_r = \frac{d\vec{V}}{dt} + 2\vec{\Omega}\times\vec{V} - \Omega^2\vec{R} \qquad \text{Thus,}$$

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega}\times\vec{V} - \frac{1}{\rho}\nabla p + g^* + \Omega^2\vec{R} + F_r \qquad \text{Since, } g = g^* + \Omega^2\vec{R}$$

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega}\times\vec{V} - \frac{1}{\rho}\nabla p + g + F_r$$

Component equations in spherical coordinates

- Expansion of the acceleration vector on a sphere gives:

$$\frac{d\vec{V}}{dt} = \hat{i}\frac{du}{dt} + \hat{j}\frac{dv}{dt} + \hat{k}\frac{dw}{dt} + u\frac{d\hat{i}}{dt} + v\frac{d\hat{j}}{dt} + w\frac{d\hat{k}}{dt} \qquad \text{(since the i, j, and k unit vectors change position)}$$

Expand the term $\frac{d\hat{j}}{dt}$ using the definition of the total derivative, and show which partial derivatives do not go to zero. Evaluate one of the terms that does not go to zero.

answer: $\frac{d\hat{j}}{dt} = \frac{\partial\hat{j}}{\partial t} + u\frac{\partial\hat{j}}{\partial x} + v\frac{\partial\hat{j}}{\partial y} + w\frac{\partial\hat{j}}{\partial z}$ *The 1st and 4th terms on the RHS go to zero.*

To evaluate the 2nd term on the RHS, draw figure 2.4 in Holton. Using similarity of triangles you can show that: $\frac{\delta\hat{j}}{\delta y} = \frac{\hat{j}}{a}$ *Take the limit as $\delta y \to 0$, use the fact that the unit vector $\hat{j}$ =1, and note that the vector $\delta\hat{j}$ is pointed downward (in the negative k direction), thus:*

$$\frac{\partial\hat{j}}{\partial y} = -\frac{\hat{k}}{a}$$

- What are the steps to find the component equation in spherical coordinates?

answer:

1) Define the coordinate system (i.e. longitude, latitude, and height denote the E/W, N/S, and up/down directions, respectively).

2) Expand the acceleration vector into components on a sphere.

3) Evaluate the terms involving total derivatives of the unit vectors i, j, and k.

4) Substitute expressions for the total derivatives of i, j, and k back into the expanded acceleration vector (expanded acceleration vector obtained from step 2).

5) Multiply through by u, v, and w and put all of the i, j, and k terms together.

6) Expand the force terms (i.e. PGF, g, and Friction) and then, equating all of the i, j, and k directions, we obtain the eastward, northward, and vertical component momentum equations respectively.

Scale analysis of the equations of motion

- Why do we use scale analysis?

answer: to simplify the math and filter out unwanted types of motion (i.e. sound waves)

- What are the forces involved in the geostrophic and hydrostatic balance approximations and where to these approximations come from?

answer: Forces in the geostrophic balance are the coriolis and horizontal pressure gradient forces which are the largest terms when scale analysis is performed on the horizontal momentum equations. Forces in the hydrostatic balance are the vertical pressure gradient and gravity force which are the largest terms when scale analysis is performed on the vertical momentum equation.

- What can be inferred regarding the direction of the geostrophic wind from the following equation:

$$\vec{V}_g = \hat{k} \times \frac{1}{\rho f} \nabla p$$

answer: The cross product tell us that the geostrophic wind blows perpendicular to the pressure gradient, or parallel to isobars.