Constant Angular momentum Oscillations

\[
\frac{\text{d}y}{\text{d}t} = -f_0 \quad \frac{\text{d}(u)}{\text{d}t} = f_v \quad \text{differentiate w/ respect to time...}
\]

\[
\frac{\text{d}^2 u}{\text{d}t^2} = f \frac{\text{d}v}{\text{d}t}
\]

Substitute \( \frac{\text{d}v}{\text{d}t} = -fu \)

\[
\frac{\text{d}^2 u}{\text{d}t^2} = -f^2 u
\]

\[
\frac{\text{d}^2 u}{\text{d}t^2} + f^2 u = 0
\]

\( r^2 + f^2 = 0 \) use quadratic formula, \( r = \frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a} \)

\[
r = \frac{0 \pm \sqrt{0 - 4f^2}}{2} = \frac{1}{2} \sqrt{-4f^2} = \pm if
\]

\( r = 0 \pm if \) thus \( \lambda = 0 \) and \( \mu = f \)

and the general solution if \( \frac{\text{d}^2 u}{\text{d}t^2} + f^2 u = 0 \) is:

\[
u(t) = c_1 e^{\lambda t} \cos \mu t + c_2 e^{\lambda t} \sin \mu t \quad \text{make substitutions}
\]

\[
u(t) = c_1 \cos ft + c_2 \sin ft
\]

Now apply initial values to find \( c_1 \) and \( c_2 \)
at time $t = 0 \implies \begin{cases} u(0) = u_0 \\ u'(0) = 0 \end{cases}$

- first substitute $u(0) = u_0$

$$u_0 = c_1 \cos f [0] + c_2 \sin f [0]$$

$$c_1 = u_0$$

- now substitute $u'(0) = 0$

$$0 = -u_0 \sin f [0] + c_2 \cos f [0]$$

$$c_2 = 0$$

Thus,

$$u(t) = u_0 \cos ft$$

to find $v(t)$ use some trig...

\[ \sin \phi = \frac{v(t)}{u_0} \]

$$v(t) = u_0 \sin \phi$$

Since $\cos \phi = \frac{u_0 \cos ft}{u_0}$

$$\phi = ft$$

\[ v(t) = u_0 \sin ft \]
• Use \( u = \frac{dx}{dt} \) and \( v = \frac{dy}{dt} \) and integrate with respect to time \( t \) to obtain the position of an object at time \( t \):

\[
\begin{align*}
\int \frac{dx}{dt} \, dt &= \int u_0 \cos ft \, dt \\
\int \frac{dy}{dt} \, dt &= \int u_0 \sin ft \, dt
\end{align*}
\]

\[
\begin{align*}
x - x_0 &= \frac{u_0}{f} \sin ft \\
y - y_0 &= \frac{u_0}{f} \cos (ft - 1)
\end{align*}
\]

\( \text{Note}: \) variation of \( f \) with latitude is neglected.

(1) and (2) show that in the northern hemisphere, the object orbits anticlockwise in a circle of radius \( R = \frac{u_0}{f} \) about the point \((x_0, y_0 - \frac{u_0}{f})\) with a period given by

\[
T = \frac{2\pi R}{u_0} = 2\pi f = \frac{\pi}{(2 \sin \phi)}
\]

Constant angular momentum oscillations are commonly observed in the oceans but are not important in the atmosphere.