Constant Angular momentum Oscillations

$\left(\frac{dv}{dt}\right)_{co} = -fu$ $\quad \left(\frac{du}{dt}\right)_{co} = fv \rightarrow$ differentiate w/ respect to time....

$\downarrow$

$\frac{d^2u}{dt^2} = f\frac{dv}{dt}$ $\quad$ substitute $\frac{dv}{dt} = -fu$

$$\frac{d^2u}{dt^2} = -f^2u$$

$\boxed{\frac{d^2u}{dt^2} + f^2u = 0}$ $\rightarrow$ solve for u; this is a 2nd order differential equation so we must find the roots of the characteristic equation

$r^2 + f^2 = 0$ $\rightarrow$ use quadratic formula, $r = \frac{-b^2 \pm \sqrt{b^2 - 4ac}}{2a}$

$$r = \frac{0 \pm \sqrt{0 - 4f^2}}{2} = \frac{\pm\sqrt{-4f^2}}{2} = \pm if$$

$r = 0 \pm if$ $\rightarrow$ thus $\lambda = 0$ and $\mu = f$

and the general solution of $\frac{d^2u}{dt^2} + f^2u = 0$ is ....

$u(t) = c_1 e^{\lambda t} \cos \mu t + c_2 e^{\lambda t} \sin \mu t$ $\rightarrow$ make substitutions

$$u(t) = c_1 \cos ft + c_2 \sin ft$$

now apply initial values to find c_1 and c_2

at time $t=0 \Rightarrow \begin{cases} u(0) = U_0 \\ u'(0) = 0 \end{cases}$

• first, substitute $U(0) = U_0$ .....

$$U_0 = C_1 \cancelto{1}{\cos f[0]} + C_2 \cancelto{0}{\sin f[0]}$$

$$C_1 = U_0$$

• now substitute $u'(0) = 0$

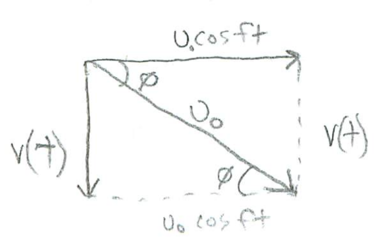
$$0 = -U_0 \sin f[0] + C_2 \cos f[0]$$

$$C_2 = 0$$

thus,

$$\boxed{u(t) = U_0 \cos ft}$$

to find $v(t)$ use some trig...



$$\sin \phi = \frac{v(t)}{U_0}$$

$$v(t) = U_0 \sin \phi$$

since $\cos \phi = \frac{\cancel{U_0} \cos ft}{\cancel{U_0}}$

$$\phi = ft$$

$$\boxed{v(t) = U_0 \sin ft}$$

• use $u = \frac{dx}{dt}$ and $v = \frac{dy}{dt}$ and integrate with respect to time to obtain the position of an object @ time $= t$

$$\int \frac{dx}{dt}\, dt = \int U_0 \cos ft \, dt \qquad \int \frac{dy}{dt}\, dt = \int U_0 \sin ft \, dt$$

$$\boxed{x - x_0 = \frac{U_0}{f} \sin ft} \quad ① \qquad \boxed{y - y_0 = \frac{U_0}{f} \cos(ft - 1)} \quad ②$$

** note: variation of f with latitude is neglected

① and ② show that in the northern hemisphere, the object orbits anticyclonically in a circle of radius $R = \frac{U_0}{f}$ about the point $(x_0, y_0 - \frac{U_0}{f})$ with a period given by

$$\tau = 2\pi R / U_0 = 2\pi f = \pi / (\Omega \sin \phi)$$

Constant angular momentum oscillations are commonly observed in the oceans but are not important in the atmosphere.