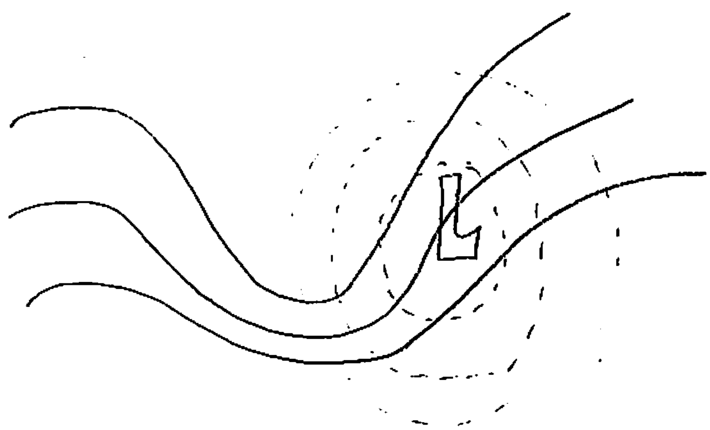
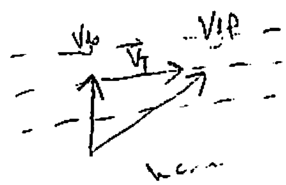
* Application of thermal wind to typical synoptic chart ....

500 mb heights / MSLP

HW →
3.10
3.11
3.20
3.21
3.23

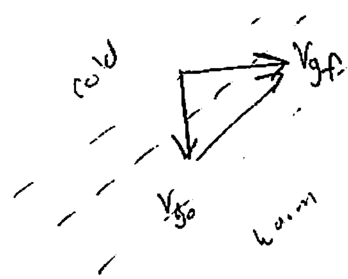


East of low



$\boxed{\text{WAA}}$

West of low



$\boxed{\text{CAA}}$

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yesterday we derived the thermal wind....

$$\vec{V}_T = \frac{R}{f}\,\hat{k} \times \nabla \langle T \rangle \ln {}^{p_0}/_{p_1} \;\longrightarrow\;$$ $\vec{V}_T$ blows parallel to isotherms w/ warm air to the right!!

Lecture 3/9/2007

* Barotropic and Baroclinic disturbances

* for a barotropic atmosphere density depends only on pressure ...

$$\rho = \rho(p)$$

* Thus, isobaric surfaces are also surfaces of constant density. We can easily infer this from the ideal gas law ...

$$p = \rho R \boxed{T}$$ — no dependence on T

* For an ideal gas the isobaric surfaces will also be isothermal if the atmosphere is barotropic.

Thus, $\nabla_p T = 0$ in a barotropic atmosphere

* So, using the thermal wind equation.

$$\frac{\partial \vec{V}_g}{\partial \ln p} = -\frac{R}{f}\hat{k}\times \overset{0}{\nabla_p T} = 0$$

we see that the geostrophic wind does not change w/ height in a barotropic atmosphere.

* This makes barotropy a very strong constraint on motions in a rotating fluid → large scale motions only depend on

* Baroclinic atmospheres have density that depends on both T and p ---

$$\rho = \rho(T, p)$$

* In a baroclinic atmosphere the geostrophic wind generally has vertical shear which is related to the horizontal temperature gradient by the thermal wind equation.

→ Obviously, baroclinic atmospheres are of primary importance in dynamic meteorology but much can be learned from the simpler barotropic atmosphere, which we'll see later ---

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3.5 Vertical Motion

* For synoptic scale motions vertical velocity is only on the scale of a few cm/s.

* Meteorological sounding only give accuracy to O~1m/s.

→ Thus, no way to measure vertical velocity directly; It must be inferred.

* 2 methods for measuring vertical velocity

1) Kinematic method
2) adiabatic method

* Both methods are applied in an isobaric coordinate system so that $\omega(p)$ is inferred rather than $w(z)$.

* However, ω and w can be related using the hydrostatic equation ....

1st → Expand total derivative ..

$$\omega = \frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}$$

these can be combined to represent horizontal advection ..

$$\omega = \frac{\partial p}{\partial t} + \vec{V}\cdot\nabla p + w\frac{\partial p}{\partial z}$$

* For synoptic scale motions, horizontal velocity is geostrophic to a first approximation (close to geostrophic but not quite)

Thus, we can express the real wind ...

$\vec{V} = \vec{V}_g + \vec{V}_a$ where $\vec{V}_a$ is "ageostrophic" wind

$$|\vec{V}_a| << |\vec{V}_g|$$

* We can substitute into "ω equation" ...

$$\omega = \frac{\partial p}{\partial t} + (\vec{V}_g + \vec{V}_a) \cdot \nabla p + w \frac{\partial p}{\partial z}$$

→ we have showed that $\vec{V}_g$ blows parallel to isobars so $\vec{V}_g \perp \nabla p$ thus ....

[substitute hydrostatic]

$$\omega = \frac{\partial p}{\partial t} + \vec{V}_a \cdot \nabla p - g\rho w$$

* We can perform scale analysis on the RHS ...

① $\frac{\partial p}{\partial t} \sim 10\ hPa/day$ [typically see 10mb p fluctuation in one day]

② $(\vec{V}_a \cdot \nabla p) \sim [1\ ms^{-1}] \cdot \left[\frac{10\ hPa}{10^6\ m}\right] \sim 10^{-5}\ hPa/s \left[\frac{60 \cdot 60 \cdot 24}{1}\right] \sim 1\ hPa/day$

③ $g\rho w \sim (10\ ms^{-2})(1\ kg/m^3)(10^{-2}\ ms^{-1})$

$\sim 10^{-1}\ kg\ m^{-1} s^{-3}$ $[1\ Pa = kg\ m^{-1} s^{-2}]$

$\sim 10^{-1}\ Pa/s \cdot \left[\frac{(60 \cdot 60 \cdot 24)}{1}\right] \cdot \left[\frac{1}{100}\right]$

$\sim 100\ hPa/day$

* Thus it is a good approximation to let ...

$$\boxed{\omega = -\rho g w}$$

Kinematic method

→ This method uses the continuity equation

→ Recall, we derived this equation in isobaric coordinates ...

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

- How can we interpret this??

* Remember, we can only directly measure horizontal derivatives of velocity components (not vertical)

* Thus, we could evaluate the LHS which is CON/DIV

→ This equation tells us that when there is CON/DIV there must be vertical acceleration

→ To extract velocity from acceleration we have to perform an integration ...

first manipulate slightly ---

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p$$

integrate from p_s to p

$$\int_{p_s}^{p} \frac{\partial \omega}{\partial p}\, dp = -\int_{p_s}^{p} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p dp$$

If we let $U = \langle U \rangle$ → average u from p_s to p

and

$V = \langle v \rangle$ → " v "
" " "

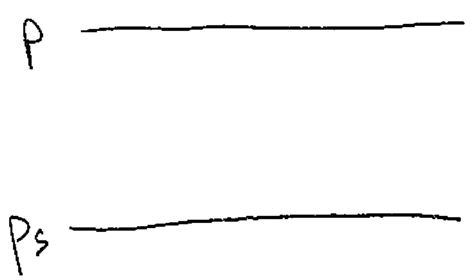
$$\omega(p) - \omega(p_s) = -(p - p_s)\left[\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right]_p$$

rearrange a little ---

$$\boxed{\omega(p) = \omega(p_s) + (p_s - p)\left[\frac{\partial \langle u \rangle}{\partial x} + \frac{\partial \langle v \rangle}{\partial y}\right]_p}$$

* We could also express this using vertical velocity in height coordinates → Holton does this ---

What does this Equation tell us???



If we know the vertical velocity @ the surface (ps) then we can estimate the vertical velocity @ p using the average horizontal DIV in the layer ps → p.

* @ the sfc $\omega = 0$

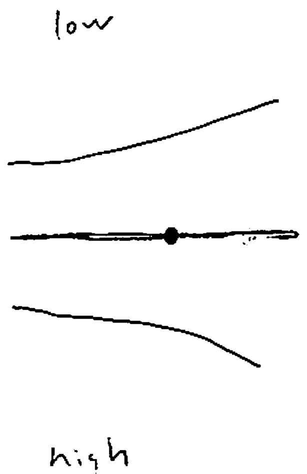
* Divergence is estimated using finite difference approximations

* There are some problems w/ using this method...

For synoptic scale motions wind is nearly in geostrophic equilibrium ... Remember, we were able to show that the divergence of $\vec{V}_g$ is zero, so that in geostrophic conditions there would be no vertical velocities.

* This implies that $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ must be nearly equal in magnitude, but <u>opposite</u> in sign.

* To prove this, consider a typical height pattern you'd see on a synoptic chart...



$\frac{\partial u}{\partial x}$ (−) $\qquad$ $\frac{\partial v}{\partial y}$ (+)

• Typically we say that this pattern implies divergence...

We would get divergence if the real wind obeys relationships we found earlier...

[i.e., $\vec{V}_g$ accurate for straight flow
$\vec{V}_g$ overestimate for cyclonic flow
$\vec{V}_g$ underestimate for anti-cyclonic flow]

→ If this is case

$$\left|\frac{\partial u}{\partial x}\right| > \left|\frac{\partial v}{\partial y}\right|$$

since $\frac{\partial u}{\partial x}$ (−) this would imply divergence.

* The horizontal divergence is due to small departures from geostrophic balance ($\vec{V}_a$) → thus a small error (~10%) could result in a large error when measuring divergence.

* Because of this, the continuity equation is not recommended for estimating vertical motion

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Adiabatic method

→ not so sensitive to errors in measured horizontal velocities.

→ Based on thermodynamic energy equation

if J is small we can write ---

$$\omega = S_p^{-1}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

unless obs are taken @ close intervals this is difficult to measure

T adv. can be measured quite accurately in mid-latitude by geostrophic winds

→ Also, this method is inaccurate when strong diabatic heating occurs, like in storms w/ heavy rain.