Review For 443 Final Exam

* Chapter 1

① Fundamental Forces

→ PGF: $-\frac{1}{\rho}\nabla p$, points from Ⓗ to Ⓛ can also express as $\nabla\Phi$ which you should be able to show mathematically

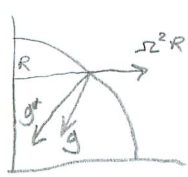
→ gravitational force: proportional to mass inversely proportional to distance between two masses

→ Friction: not important for large scale motions

② Apparent Forces

→ centrifugal force: how is it different from centripetal acceleration? why must we introduce it in a coordinate system rotating w/ Earth

→ gravity force: gravity plus centrifugal acceleration



- what is the difference between g and g* ?
- why is g* constant for meteorology applications?

→ Coriolis force: deflection of moving objects caused by Earth's rotation

$$\left(\frac{du}{dt}\right)_{co} = 2\Omega v \sin\phi = fv$$

$f = 2\Omega \sin\phi$ = coriolis parameter

$$\left(\frac{dv}{dt}\right)_{co} = 2\Omega u \sin\phi = -fu$$

③ Hydrostatic Balance

→ what is it $\frac{\partial p}{\partial z} = -\rho g$ → $-\frac{1}{\rho}\frac{\partial p}{\partial z} = g$ (force balance)

→ The hydrostatic equation can be used to derive the hypsometric equation

$$Z_T = \frac{R\langle T \rangle}{g_0} \ln P_1/P_2$$ → what does this tell us about the atmosphere??

* Chapter 2

① Total differentiation

→ what is the difference b/w $\frac{d()}{dt}$ and $\frac{\partial ()}{\partial t}$?

→ Expand $\frac{d()}{dt}$ → $$\frac{d()}{dt} = \frac{\partial ()}{\partial t} + \underbrace{u\frac{\partial ()}{\partial x} + v\frac{\partial ()}{\partial y} + w\frac{\partial ()}{\partial z}}$$

what do these terms represent??

② Component Equations in spherical coordinates

→ Vectorial form of the momentum equation in spherical coordinates is:

$$\frac{d\vec{V}}{dt} = -2\vec{\Omega}\times\vec{V} - \frac{1}{\rho}\nabla p + g + F_r$$

→ It is useful to break down the above equation into it's components on a sphere.

- Why? → useful for numerical wx prediction and theoretical analysis
- How? →

1. Expand acceleration term ...

$$\frac{d\vec{V}}{dt} = \frac{du}{dt}\hat{i} + \frac{dv}{dt}\hat{j} + \frac{dw}{dt}\hat{k} + \frac{d\hat{i}}{dt}u + \frac{d\hat{j}}{dt}v + \frac{d\vec{k}}{dt}w$$

and evaluate $\frac{d\hat{i}}{dt}$, $\frac{d\hat{j}}{dt}$, and $\frac{d\vec{k}}{dt}$

2. Find the components of the "force terms" and combine all terms acting in each direction.

→ The component equations in spherical coordinates are:

$$\frac{du}{dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + F_{rx}$$

$$\frac{dv}{dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\phi + F_{ry}$$

$$\frac{dw}{dt} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\phi + F_{rz}$$

* Be able to identify all terms ---

(3) Scale Analysis → technique for estimating magnitudes of terms in the governing equations for a particular type of motion.

→ why do we perform scale analysis?
simplify math and filter out unwanted types of motion (e.g. sound waves)

→ should know characteristic scales of field variables for synoptic scale motions ---

$U \sim 10\ ms^{-1}$ $\qquad H \sim 10^4\ m$

$W \sim 10^{-2}\ ms^{-1}$ $\qquad \delta p \sim 10\ hPa$

$L \sim 10^6\ m$ $\qquad L/U \sim 10^5\ s \sim 1$ day

→ what do we get when we perform scale analysis on the horizontal momentum equations and keep the highest order terms??

$$\left.\begin{aligned} -fv &\approx -\frac{1}{\rho}\frac{\partial p}{\partial x} \\ fu &\approx -\frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned}\right\} \rightarrow \text{geostrophic relationship}$$

geostrophic wind can be defined as $\boxed{\vec{V}_g = -\vec{k} \times \frac{1}{\rho f}\nabla p}$

what does this eq. tell about geostrophic wind direction??

→ what do we get when we perform scale analysis on vertical momentum equation?

$$g = -\frac{1}{\rho}\frac{\partial p}{\partial z} \quad \text{or} \quad \frac{\partial p}{\partial z} = -\rho g \rightarrow \text{hydrostatic balance}$$

→ what if we keep highest and 2nd highest terms in horizontal momentum equation?

$$\left.\begin{aligned} \frac{du}{dt} &= fv - \frac{1}{\rho}\frac{\partial p}{\partial x} \\ \frac{dv}{dt} &= -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} \end{aligned}\right\} \rightarrow \text{prognostic equations}$$

→ How do we know if geostrophic approximation is accurate?

$$R_o = \frac{ACC}{COR} = \frac{U^2/L}{f_0 U} = \frac{U}{f_0 L}$$

When ACC is small compared to COR (small R_o) the geostrophic approximation is valid w/ only small errors.

(4) → Continuity Equation → Expresses conservation of mass

- One form is:

$$\frac{d\rho}{dt} + \nabla \cdot (\rho \vec{V}) = 0 \qquad (1)$$

- The other form is

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot \vec{V} = 0 \qquad (2)$$

* Should be able to interpret both forms and get from one form to the other.
* What happens to 2nd eq. when fluid is incompressible??

5. Thermodynamics of the Dry atmosphere ...

→ We derived potential temperature

$$\Theta = T\left(\frac{P_s}{P}\right)^{R/c_p} \rightarrow \text{what is it?}$$

→ Can you use Θ to derive adiabatic lapse rate??

→ Static stability → you should be able to specify type of stability for different types of Θ and T profiles.

* chapter 3

1. Conversion of Basic Equations to isobaric coordinates ...

* What are disadvantages and advantages for conversion of the:

  momentum equation
  continuity equation
  thermodynamic equation

2. natural coordinates → $\hat{t}$ is unit vector parallel to motion
   $\hat{n}$ is " " normal "

* We showed that:

$$\frac{dV}{dt} = -\frac{\partial \Phi}{\partial s} \quad (\text{in } \hat{t}\text{-direction})$$

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n} \quad (\text{in } \hat{n}\text{-direction})$$

→ Explain and interpret these equations

③ Balanced Flow

* Explain the conditions necessary for the different types of force balances ---

→ geostrophic balance

→ inertial flow

→ cyclostrophic balance

④ Gradient wind approximation: Balance b/w COR, PGF, and centrifugal

* Use quadratic formula to solve for V ---

$$V = -\frac{fR}{2} \pm \left(\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}\right)^{1/2}$$

* How do we obtain physical solutions from this equation?

V ⊕ and no imaginary #s

* what are the 4 physical solutions and the signs of R and $\frac{\partial\Phi}{\partial n}$ for these solutions?

* Why is the wind speed limited in high pressure regions?

* For what type of flow is V_g an over and underestimate of real wind?

5 Trajectories and streamlines

* Trajectories trace path of parcel
* streamlines are parallel to instantaneous velocity field

why does using $\nabla\Phi$ to estimate radius of curvature in gradient wind approximation lead to errors.

6 Thermal wind

→ Thermal wind equation ⇒ $\frac{\partial \vec{V}_g}{\partial p} = -\frac{R}{f}\hat{k}\times\nabla_p T$

what does this equation tell us? The geostrophic wind must have vertical shear in the presence of a horizontal temperature gradient.

* we could also write:

$$\vec{V}_T = \vec{V}_g(p_1) - \vec{V}_g(p_0) = \frac{R}{f}\hat{k}\times\nabla\langle T\rangle \ln p_0/p_1$$

→ Use this eq. to show how you can find the T-adv. from a single sounding.

7 Vertical motion

→ Be able to express ω in terms of w

→ we discussed 2 methods for estimating vertical motion:

1 Kinematic method → $\omega(p) = \omega(p_s) + (p_s - p)\left[\frac{\partial\langle u\rangle}{\partial x} + \frac{\partial\langle v\rangle}{\partial y}\right]$

2 adiabatic method → $\omega = S_p^{-1}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$

what are problems associated with using each method??

⑧ Surface pressure tendency

$$\frac{\partial p_s}{\partial t} = -\int_0^{p_s} (\nabla \cdot \vec{V})\, dp \rightarrow \text{interpret this equation}$$

Be able to apply this equation to explain "heat low" formation.

* Chapter 4

① What are differences b/w circulation and vorticity?

Circulation: scalar integral; macroscopic measure of rotation over a finite area

Vorticity: vector that gives microscopic measure of rotation @ any point in fluid

② Circulation is expressed mathematically as:

$$C = \oint \vec{V} \cdot d\vec{l} \rightarrow \text{interpret this equation}$$

"line integral evaluated along a closed contour of the velocity component parallel to the contour"

③ Kelvin's circulation theorem:

$$\frac{d}{dt}(C_a) = -\oint \frac{1}{\rho}\, dp \quad \text{interpret this ...}$$

"rate of change in absolute circulation following the motions results from intersection of p and ρ surfaces"

④ Vorticity → 3-d vector

$$\vec{\omega} = \underbrace{\nabla \times \vec{V}}_{} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} , \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} , \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

"curl" of
velocity
vector

$$\zeta = \hat{k} \cdot (\nabla \times \vec{V}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

relative vorticity is vertical component of 3-d vorticity vector

⑤ What is the relationship between circulation and vorticity?

$$\zeta = \frac{C}{A} \quad \text{or} \quad \zeta = \lim_{A \to 0} \left(\oint \vec{V} \cdot dl \right) A^{-1}$$

⑥ In natural coordinates, vorticity can be expressed as

$$\zeta = -\frac{\partial V}{\partial n} + \frac{V}{R_s}$$ → what do these terms represent??

→ How can we get vorticity when flow is straight?

→ How can we get <u>no</u> vorticity when flow is curved?

7 Potential Vorticity → "PV" is conserved following the motion for adiabatic frictionless flow.

$$PV = (\zeta_\theta + f)\left(-g\frac{d\theta}{dp}\right) = \text{const.}$$

→ Isentropic form of Ertel's PV

→ One way to think of PV: ratio of abs. vorticity to depth of vortex; In this case the depth is the distance b/w θ surfaces in terms of p-units.

→ Important concept: This equation implies that a parcel of air acquires more cyclonic vorticity as it moves into a region of lower static stability.

→ How is PV distributed in the vertical?

→ How do we diagnose the "dynamic tropopause"?

→ What variable is conserved on the dynamic tropopause?

→ Why is a dynamic tropopause analysis useful?

→ If we analyzed PV on a 300 mb chart, what would values of PV above 3 PVUs imply??

→ For a homogeneous incompressible fluid, PV can be simplified to...

$$\eta / h = \text{const.}$$

* If h is constant, abs. vorticity, η, is conserved.

* Be able to show that η is conserved for northward and southward curving Easterly flow but <u>not</u> conserved for northward and southward curving westerly flow

* Using PV arguments be able to explain what type of path air parcels should follow when they go over a topographic barrier from the west and from the East.

⑧ The vorticity Equation

* How do we derive it ??

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial v}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}\right] \quad ①$$

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}\right] \quad ②$$

then take ② − ①

(9) The vorticity equation in height coordinates is —

$$\frac{d}{dt}(\zeta+f) = -(\zeta+f)\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right] - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Be able to identify and explain all terms ---

(10) The vorticity equation in isobaric coordinates can be expressed as ---

$$\frac{\partial \zeta}{\partial t} = -\vec{V}\cdot\nabla(\zeta+f) - \omega\frac{\partial \zeta}{\partial p} - (\zeta+f)\nabla\cdot\vec{V} + \vec{k}\cdot\left(\frac{\partial \vec{V}}{\partial p}\times\nabla\omega\right)$$

Be able to identify and explain all terms ---

What happened to the solenoidal term in the p-coordinate form?

What are the important terms for synoptic scale motions?