Meteorology 443

Mid-Term Test
17 April 2002
Closed-book test: 50 minutes

I. The thermodynamic equation is

\[ c_p \frac{dT}{dt} - \alpha \frac{dp}{dt} = \dot{q}. \]

5
a. Use this equation to find the relationship between \( \theta \) and \( T \). Clearly state your assumption to form this relationship.

4
b. Based on the relationship between \( \theta \) and \( T \), one can obtain static stability as the following equation:

\[ \frac{1}{\theta} \frac{\partial \theta}{\partial z} = \Gamma_d - \Gamma, \]

where \( \Gamma_d \) and \( \Gamma \) are dry adiabatic lapse rate and environmental lapse rate, respectively. Use the skew – T chart to explain the condition with which the atmospheric environment becomes convectively unstable.

II. The balanced flow without any forcing along geopotential height contours can be depicted by the equation:

\[ \frac{\nu^2}{R} + f v = -\frac{\partial \phi}{\partial n}. \]

4
a. What is the difference between gradient (\( \vec{v}_g \)) and geostrophic wind (\( \vec{v}_g \))? Use this equation to explain your answer, but you have to use the height contour structure to explain your answer.

4
b. Under what synoptic condition is \( |\vec{v}_g| > |\vec{v}_g| \) (\( \gamma \)). Explain your answer.

5
c. Use the gradient wind equation to explain why a regular low system has strong wind.

III. The thermal wind equation is

\[ \frac{\partial \vec{v}_g}{\partial \ln p} = -\frac{R}{f} \hat{k} \times \nabla_p T. \]
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4  a. Use the thermal wind equation to explain why the maximum zonal wind (jet stream) exists in midlatitudes.

4  III. b. Why is the direction of thermal wind in the Southern Hemisphere against isothermals? Use the thermal wind equation to explain your answer.

10  IV. Circulation theorem vs. vorticity equation

Bjerknes circulation theorem is

\[ \frac{dC}{dt} = \frac{\int dp}{\rho} - 2\Omega \frac{dA_e}{dt}, \]

Note that \( C = \xi A \) if \( A \) is small. Show that this circulation theorem is equivalent to the following Vorticity equation:

\[ \frac{d\xi}{dt} = -v\beta - (\xi + f) (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + \frac{1}{\rho^2} \left( \frac{\partial p}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial p}{\partial x} \right). \]

8  V. The surface pressure tendency equation

\[ \frac{\partial p_s}{\partial t} = -\int_{\partial P} (\nabla \cdot \vec{v}) dp \]

is often used to explain the following observation:

"The development of a negative surface pressure tendency is a classic warning of an approaching cyclonic weather disturbance." The simplified vorticity equation for synoptic motions is

\[ \frac{d}{dt} (\xi + f) = -(\xi + f) (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}). \]

Use this simplified vorticity equation to explain the aforementioned observation of the weather development caused by a cyclone disturbance.