Other Applications of Quasigeostrophic Theory

Trenberth formulation, Q-vectors…

Meteorology 411 – Iowa State University – Week 8

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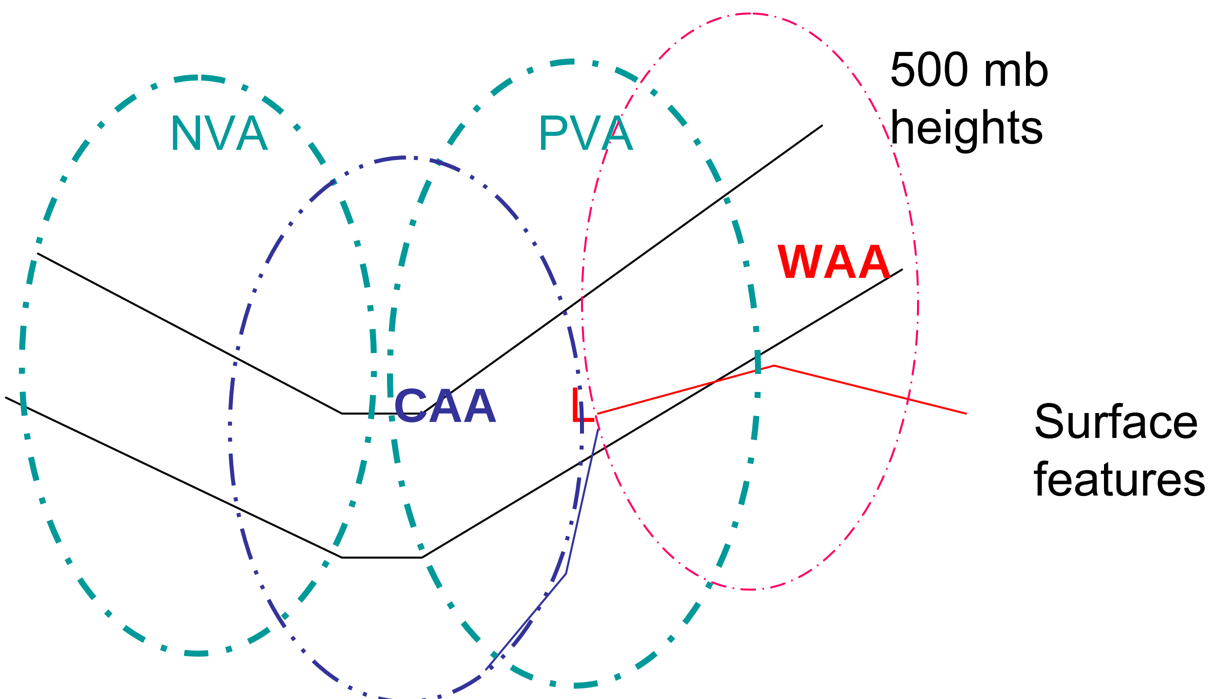
Problems with current QGω equation

- The two main terms in it, differential vorticity advection and temperature advection, often act opposite to each other



From that map…

- Notice that we have areas with both PVA and CAA (typically just behind sfc low). What would be the vertical motion there? We can't really know, and it is dangerous to "eyeball" it since remember the actual equation is not a simple function of the advections.
- We also routinely have areas of NVA and WAA – same problem.

Ways around the problem…

- We have two methods to get around this problem
- 1) Trenberth formulation of QG-omega equation
- 2) Q-vectors

Trenberth formulation of QG ω Equation

Derived by rewriting vort. Advection term to make use of thermal wind, and introducing definition of thermal deformation. This eventually yields…

$$(\nabla^2 + f_0^2/\sigma\ \partial^2/\partial p^2)\omega = f_0/\sigma\ [2(\partial \mathbf{V}_g/\partial p \nabla \zeta_g) + \partial \mathbf{V}_g/\partial p \overline{\nabla} f - 2D^2 \partial\theta/\partial p]$$

Advection of earth's vorticity by thermal wind

Deformation and thermal deformation

Advection of geostrophic vorticity by the thermal wind

Usually, the deformation term is small compared to the first term on RHS, and can be ignored.

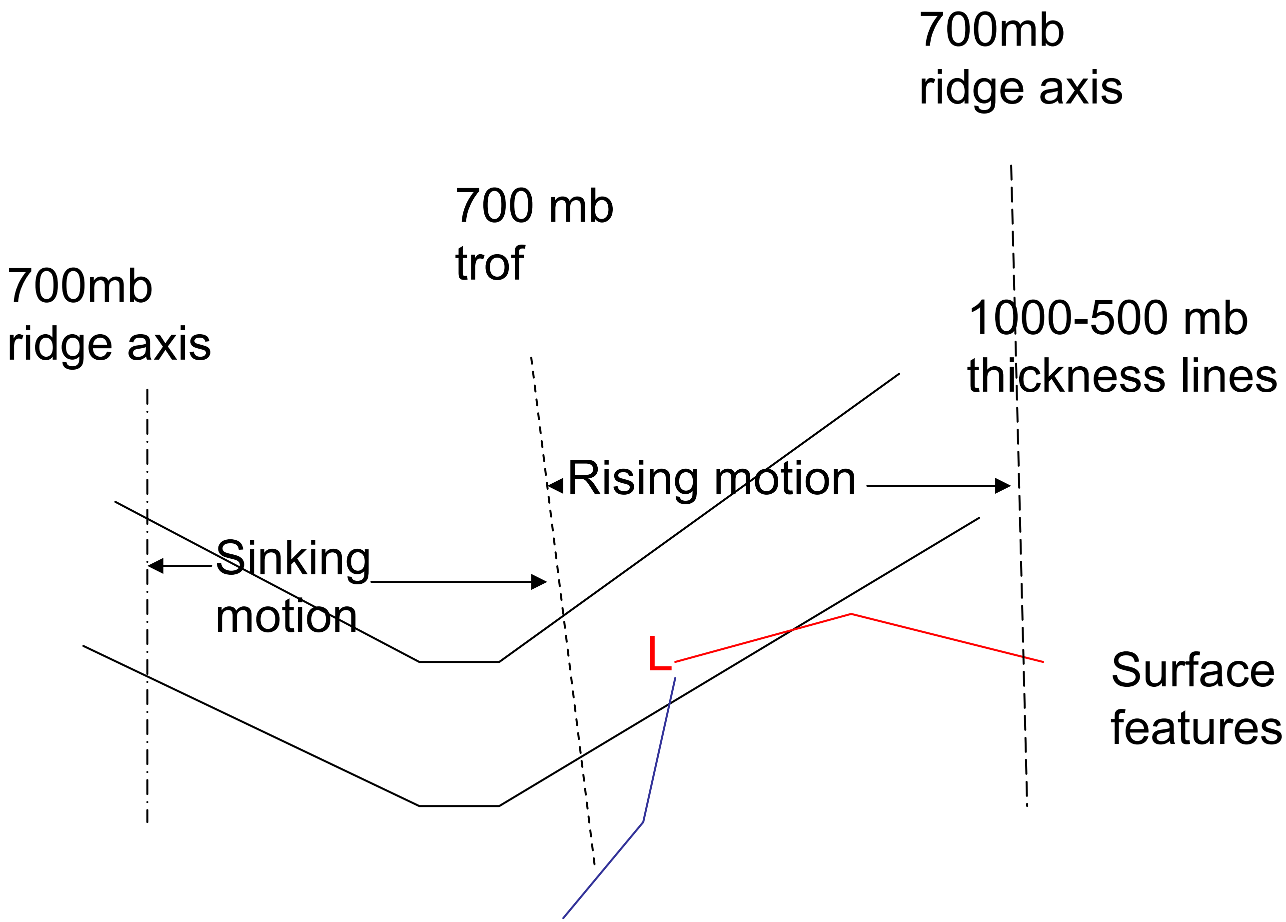
Also, $\partial \mathbf{V}g/\partial p$ is usually mostly westerly, and since gradient of f is in north-south direction, this term is also often small.

Trenberth QG ω Equation

$(\nabla^2 + f_0^2/\sigma\ \partial^2/\partial p^2)\omega = 2f_0/\sigma\ (\partial V_g/\partial p \nabla \zeta_g)$

So.. If we plot thickness, this shows us direction of thermal wind, and we can then look at some height level within that thickness layer (e.g. use 700 mb for 1000-500 mb thickness), we can see how vorticity is advected and diagnose ω

MAP INTERPRETATION



Q-vectors

- This derivation starts with the original QG equations of motion.
- Using thermal wind relation and QG Thermodynamic equation, eventually we can derive the following equation:

This term is usually ignored

$$(\nabla^2 + f_0^2/\sigma\ \partial^2/\partial p^2)\omega = -2\nabla \mathbf{Q} - R/\sigma p\beta\ \partial T/\partial x$$

$$\text{Where } \mathbf{Q} = -R/\sigma p(\partial V_g/\partial x \bar{\nabla} T) = (Q_1)$$

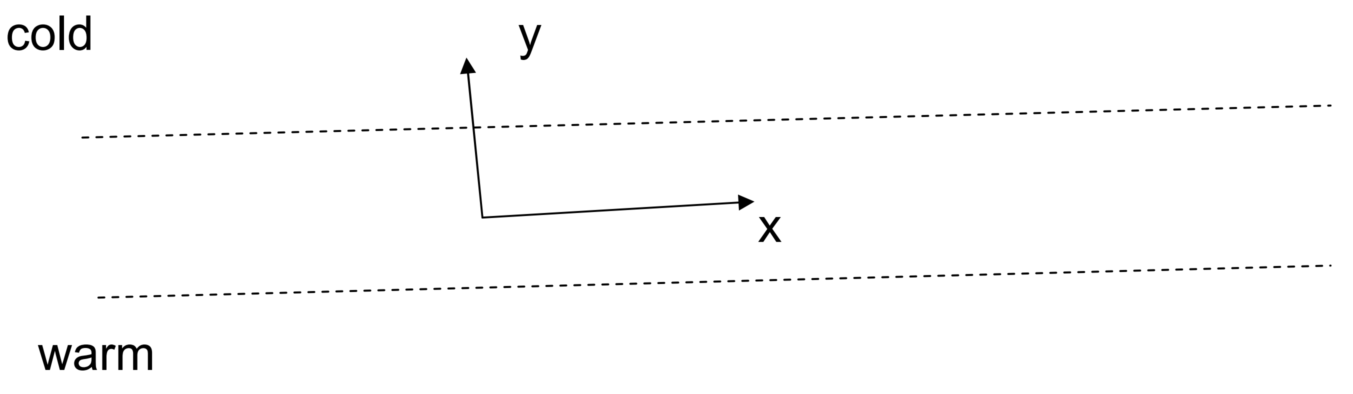
$$(\partial V_g/\partial y \bar{\nabla} T) \quad (Q_2)$$

Nice thing about this is that nothing was neglected with no extra assumptions like those needed with Trenberth formulation, AND we eliminate the problem of two opposing terms

Q vectors can be hard to compute by hand, but are easily plotted on computers. **Q-vector convergence implies upward motion**

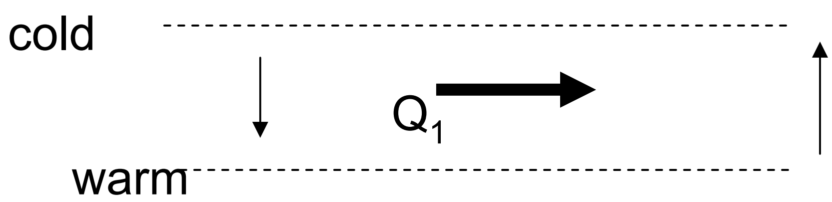
Interpretation of Q vectors

- To interpret Q-vectors, use a natural coordinate system where the x-axis is parallel to the isotherms with cold air to the left.   Y-axis is in the direction of $-\nabla T$.



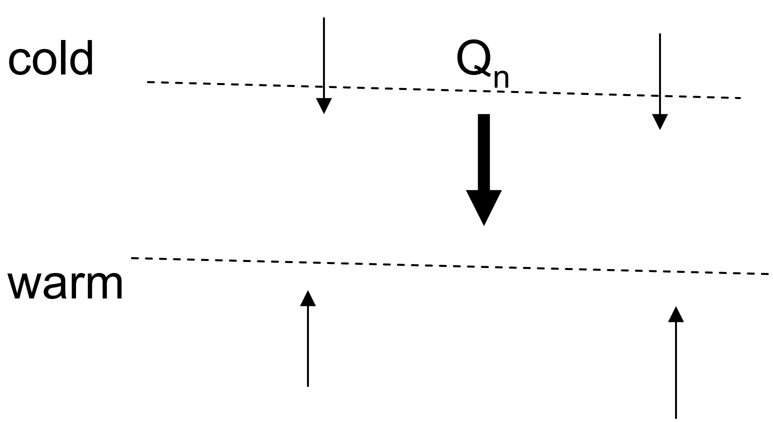
Interpretation (cont)

- $\mathbf{Q} = -R/\sigma p \; |\partial T/\partial y| \; \mathbf{k} \times \partial \mathbf{V}_g/\partial x$
- $Q_1 = -R/\sigma p \; \partial V_g/\partial x \; \partial T/\partial y = Q_s$



This Q component acts along the isotherms, and scenario is like you'd find near low pressure

- $Q_2 = -R/\sigma p \; \partial V_g/\partial y \; \partial T/\partial y = Q_n$



This Q component acts perpendicular to isotherms, in situations like a front

Interpretation (cont)

- $\nabla \mathbf{Q} > 0 \rightarrow$ sinking motion
- $\nabla \mathbf{Q} < 0 \rightarrow$ rising motion

You can imagine from the previous slide that if Q vectors point east (more generally frontward) in the center of cyclones and point west (backwards) in the center of anticyclones, the Q-vector convergence happens ahead of the cyclone, and that is where we expect ascent. Q-vector divergence happens behind cyclones, and thus downward motion.

Also, with Q-vectors pointing south (or toward warm air) in the vicinity of fronts, the Q-vector convergence will happen on the warm side of fronts, with rising motion there, and Q-vector divergence and sinking on the cold side of the front.

- Looking at the Q vector components in more detail, we see…

- If Qn points from cold to warm air, we have frontogenesis (front will form or strengthen)

Comparison of 2 techniques

- 1) Trenberth formulation involves only one forcing function which can be computed from weather maps at only one level (e.g. advection of 700 mb vorticity by 1000-500 mb thermal wind)
- 2) Q-vectors and $\nabla\mathbf{Q}$ can be estimated visually
- 3) Originally, we used 500 mb map for vorticity advection, 700 or 850 mb maps for temperature advection, and satellite for diabatic heating (deep convective areas)

Final thoughts

- Some forecast offices like to use F-vectors. These are computed like Q-vectors but they use the full wind and not the geostrophic wind. If you really think about it, though, the conclusions of QG theory were based on assumptions of things being hydrostatic and quasi-geostrophic, so it is questionable if F-vectors should really work to imply upward or downward motion.