Quasigeostrophic Theory

Quasigeostrophic Omega Equation

Meteorology 411 – Iowa State University – Week 7

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QGω Equation

- Instead of eliminating ω from our 2 equations with 2 unknowns to get the equation for X (QG Height Tendency), we can instead eliminate X to get an equation for ω
- To do this, take $R/p\sigma\ \nabla^2(2) - (f0/\sigma\ \partial/\partial p)(1)$

- $1/f_0 \nabla^2 X = -\mathbf{V}_g \nabla(\zeta_g + f) + f_0 \partial\omega/\partial p$ (Eq. 1)
- $-P/R \partial X/\partial p = -\mathbf{V}_g \nabla T + \omega\sigma P/R$ (Eq. 2)

QG ω Equation

$(\nabla^2 + f_0^2/\sigma\ \partial^2/\partial p^2)\omega = -f_0/\sigma\ \partial/\partial p[-\mathbf{V}_g\nabla(\zeta_g+f)]$
$-R/\sigma p\ \bar{\nabla}^2(-\mathbf{V}_g\bar{\nabla}T) + f_0\ \partial/\partial p(K\ \zeta_g) -$
$R/\sigma p\bar{\nabla}^2(1/c_p dQ/dt)$

Again, the term in front of ω acts like a minus sign.

QG ω Equation

Term A is the differential vorticity advection (how it changes with height)

$$(\nabla^2 + f_0^2/\sigma\ \partial^2/\partial p^2)\omega = -f_0/\sigma\ \partial/\partial p[-\mathbf{V}_g\nabla(\zeta_g+f)]$$

$$-R/\sigma p\ \nabla^2(-\mathbf{V}_g\nabla T) + f_0\ \partial/\partial p(K\ \zeta_g) -$$

Term B is quasi-horizontal Laplacian of temperature advection

Term C is change of friction with height

$$R/\sigma p\bar{\nabla}^2(1/c_p dQ/dt)$$

Term D is quasi-horizontal Laplacian of diabatic heating

Notes

- $1/\sigma$ in all terms means the more unstable the atmosphere (small σ), the greater the forcing effect (bigger vertical motions)
- $1/p$ in the temperature terms means the forcing is more efficient aloft.

Mathematical Interpretations

- Term A: Vort advection increasing with height – normally vorticity advection is bigger aloft due to stronger winds and less "closed off" systems. So, when we say PVA, we usually mean more PVA aloft than down below. In our equation, this term is $-\omega = -f_0/\sigma\ \partial/\partial p[\underline{-\mathbf{V}_g \nabla(\zeta_g+f)}]$, but we +

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  have more PVA aloft (lower pressure) than down below, so ω must be negative (upward motion)

Mathematical Interpretations

- Thus, PVA is associated with rising motion, and NVA with sinking
- Recall that the 2nd derivative of cosine or sine is –sine or –cosine. Thus, in term B, if we have warm advection, we have $\bar{\nabla}^2(+) = -$, but the negative in front makes the whole term +. So, $-\omega=+$ and thus $\omega<0$ and warm air advection is associated with rising motion, with cold air advection associated with sinking. (Note: this is technically only true if advection fields are sinusoidal)

Mathematical Interpretations

- Friction term shows for constant vorticity, since K (friction coefficient) is strongest near the ground (higher pressure), then $\partial K/\partial p>0$, so for cyclonic vorticity, $\omega<0$ meaning we have upward motion
- Diabatic heating term works like the temp. advection term. Diabatic warming gives upward motion, cooling gives downward motion

Physical Interpretation (VERY IMPORTANT)

- Differential Vorticity Advection: If the term is positive, then $\partial/\partial t(\zeta_{gA} - \zeta_{gB}) > 0$, thus over time, the vorticity aloft (A) will get more and more positive compared to that below (B)
- We can call the term $(\zeta_{gA} - \zeta_{gB})$ thermal vorticity and refer to it as ζ_T
- Based on the definition of geostrophic vorticity, $\zeta_g = 1/f_0 \nabla^2 \Phi$, we know $\partial \zeta_T / \partial t > 0$ implies …

- …..$\partial/\partial t(g/f_0\nabla^2(Z_A-Z_B)) > 0$, so this means that the thickness (Z_A-Z_B) must decrease since the Laplacian acts like a minus sign.
- So, an increase in thermal vorticity due to PVA must be associated with a thickness decrease. If the atmos. remains hydrostatic, the temperature must thus fall. If things are adiabatic, this cooling can only happen via adiabatic cooling from rising motion. So, we've proven PVA gives rising motion.

LeChatelier’s Principle

- There is a negative feedback, however, called the SECONDARY CIRCULATION, which obeys LeChatelier’s principle (remember from Chemistry?) This principle says the atmosphere acts to restore itself to equilibrium when it is disturbed, by reducing the effect of the disturbance.

Le Chatelier's principle applied to PVA

- For PVA, we found we get upward motion, but to get upward motion, we need low-level convergence and upper-level divergence. The Coriolis force acts on the div/conv to turn divergence into anticyclonic flow, and convergence into cyclonic flow. This would make the thermal vorticity more anticyclonic, directly OPPOSITE to what PVA tries to do.

Temperature Advection

- Warm air advection increases the temperature and the thickness, so

  $\partial/\partial t(Z_A-Z_B) > 0$, and thus $\bar{\nabla}^2(Z_A-Z_B) < 0$

So the thermal vorticity must be decreasing with time. This means we are getting more anticyclonic vorticity aloft, and cyclonic vort below. To do this, we need divergence aloft/convergence below (which Coriolis will act on to create the vorticity). This div/conv combo gives us upward motion.

LeChatelier’s principle for temp advection

- Upward motion will give us adiabatic cooling, which is OPPOSITE to the warm advection taking place.

- We can apply the same logic to friction and diabatic heating terms

Notes

- The primary and secondary circulations do not have a time lag between them and happen instantaneously. The ageostrophic response is instantaneous keeping the atmosphere hydrostatic and mostly geostrophic (remember the secondary circulation is the vertical motion and ageostrophic winds)

Funny thing to think about…

- Downslope gives adiabatic warming and thus a thickness increase, meaning anticyclonic tendency to thermal vorticity, which requires convergence below and divergence aloft. But what do these give us? Rising motion. Thus, downward motion (primary circ) gives upward motion (secondary)!

Quantitative Vertical Motion

- Remember we can compute vertical motion quantitatively using continuity equation:
- $\partial u/\partial x + \partial v/\partial y = -\ \partial \omega/\partial p$. If we integrate both sides in vertical, we get
- $\omega(z_1) = \omega(z_0) + \Sigma\ \overline{\mathrm{DIV}}\ \Delta p$

Roughly 0

- E.g., if ave divergence in 1000-850 mb layer is $-2\times10^{-5}\ s^{-1}$, $\omega(850) = \cancel{\omega(1000)} +$

$\Sigma\ (-2\times10^{-5}\ s^{-1})150\text{mb} = -3\ \mu b/s$ or $3\ cm/s$