Hydrostatic Equation and Thermal Wind

Meteorology 411 – Iowa State University – Week 5

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Hydrostatic Equation

- In the atmosphere, vertical accelerations (dW/dt) are normally fairly small, and we can ignore them on the synoptic scale
- If we do this, our vertical momentum equation becomes
- $\partial p / \partial z = -\rho g$ where ρ is the air density
- This is really just telling us the upward directed pressure gradient force is balanced by the downward directed force of gravity

What does the hydrostatic equation tell us?

- Therefore, pressure at any point is just the weight of the column of air above the point:
- $P = \int \rho g \, dz$ where we integrate from $z=z(p)$ to $z=$infinity
- We can also use the hydrostatic equation to estimate the mean sea level pressure

Hypsometric Equation

Use the ideal gas law $p = \rho R_d T$ and substitute for ρ in the hydrostatic equation

$dz = R_d T/g\ \partial p/p$

This can be integrated from z_1 to z_2

$z_2 - z_1 = R_d \overline{T}/g \ln(p_1/p_2)$ where p1 is the pressure at height z_1, and p_2 at z_2, and the $\overline{T}$ is the average temp in the layer.

The $z_2 - z_1$ is the thickness between levels p1 and p2.

What is thickness?

- Note that it is proportional to the average temperature in the layer. Thus, thickness increases as temperature increases, meaning the distance between two given pressure levels increases, which makes sense since the density drops as air warms up, so you'd have to have a thicker layer to contain the same amount of air
- To include moisture effects, use $\overline{T}_v$ instead of $\overline{T}$

Thermal Wind

- We’ve seen previously that one can relate the pressure/height field to winds
- Can one relate temperature to winds?
- Thermal wind shows the impact of temperature on the winds, but it itself is not really a wind

How to derive thermal wind equations

Start with geostrophic wind equations

$u_g = -1/f\ \partial\varphi/\ \partial y \quad v_g = 1/f\ \partial\varphi/\ \partial x$

Use hydrostatic equation, writing it as

$\partial\varphi/\ \partial p = -RT/p$

Substitute to get: $-\ \partial u_g/\ \partial p = -R/(fp)(\partial T/\partial y)$

$-\ \partial v_g/\ \partial p = R/(fp)(\partial T/\partial x)$

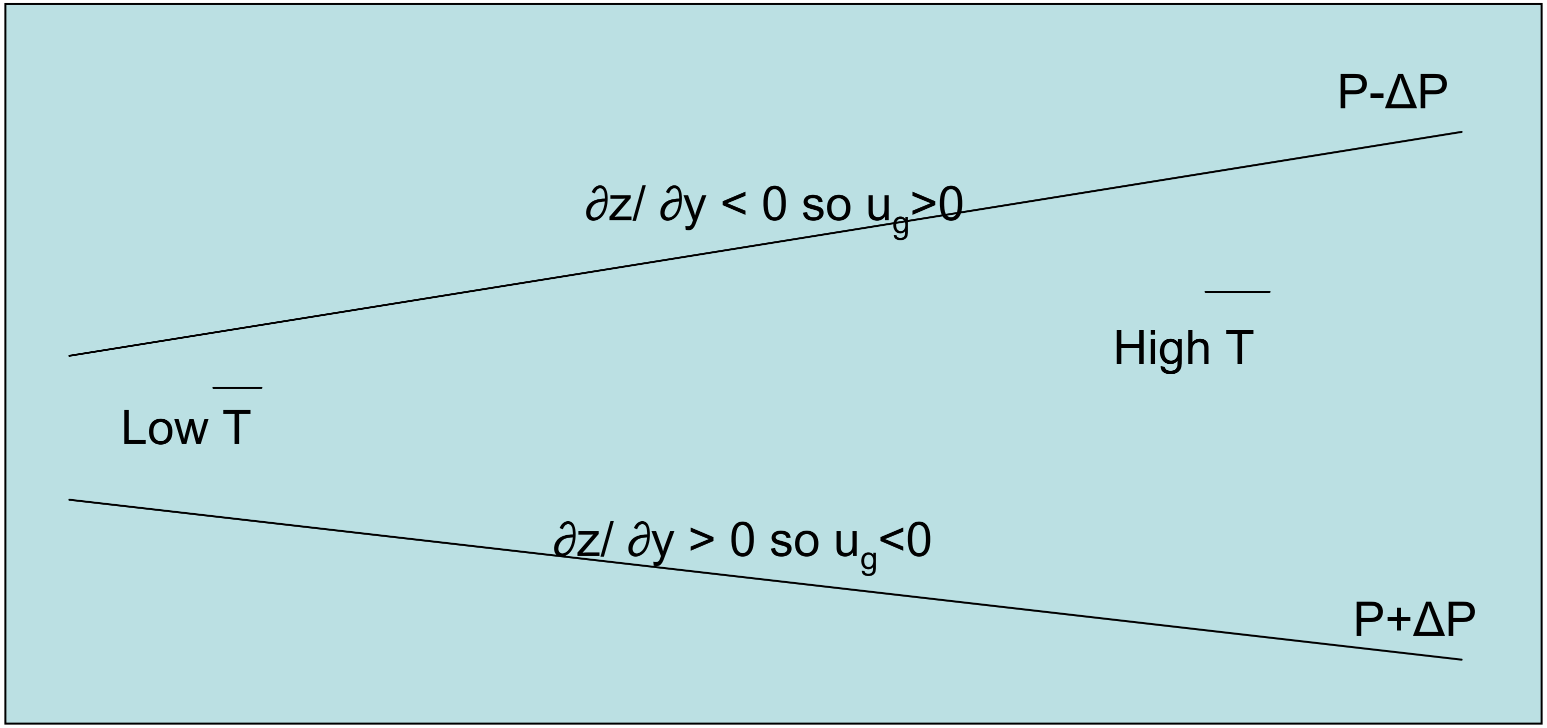
Or in vector form, $\underline{-\ \partial \mathbf{V}_g/\ \partial p} = R/(fp)(\mathbf{k}\times\nabla T)$

Thermal wind shear

Definition

- The vectorial difference between the geostrophic wind at a given pressure and the geostrophic wind at a higher pressure (lower height) is called the THERMAL WIND. The thermal wind shear vector is parallel to the isotherms/thickness lines with colder air to its left in the N. Hemisphere.

Why should geostrophic wind change if there is a horizontal temp gradient?



If the atmosphere is hydrostatic, and there is a horizontal temperature gradient, then there must be a horizontal thickness gradient, which means the p-surfaces won’t be parallel, and will have different slopes, so that the geostrophic wind will differ on the two p-surfaces.

Thermal wind equation derived from hypsometric equation

- The hypsometric equation can be written as
- $\Phi_2 - \Phi_1 = R\overline{T} \ln (p_1/p_2)$ so
- $v_{g2} - v_{g1} = g/f\ \mathbf{k} \times \nabla (z_2 - z_1) =$
  $R/f \ln (p_1/p_2) (\mathbf{k} \times \nabla \overline{T})$ so that the thermal wind is perpendicular and to the left of the mean temp. gradient (thickness), and thus parallel to thickness contours with higher values to the right

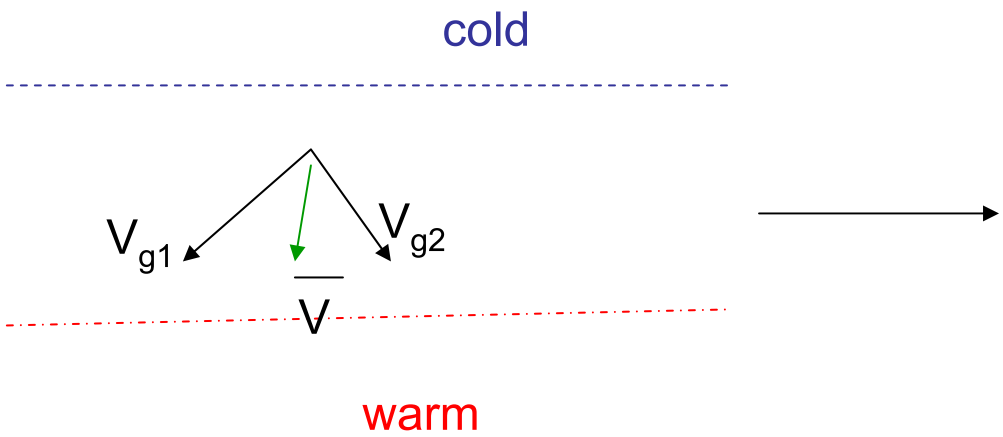
Veering and backing…

- A common forecasting rule mentions veering and backing of wind with height (veering = bad weather, backing = good).
- Technically the proof of why the rule works is only valid for the backing or veering of the GEOSTROPHIC wind with height.

Veering and backing…

- If the geostrophic wind veers with height (turns clockwise), there is mean geostrophic WARM advection in the layer
- If the geostrophic wind backs with height (turns counterclockwise), there is COLD advection.

Proof



$V_{g2} - V_{g1}$ would be this vector pointed east

This implies that the temperature contours are parallel to that vector (which is the thermal wind vector), and the cold air is to the left, or north.

The mean wind vector (green vector which is the average of the two shown) would be something pointing toward the south, which clearly implies cold advection. That is what we expected since our geostrophic winds were BACKING with height.