Dynamics of the Wind Field

Balanced Wind Approximations

***Meteorology 411 – Iowa State University – Week 4**

Bill Gallus*

Why use balanced wind approximations?

- Real atmosphere is very complex, making it hard to understand what processes are truly important
- Approximations can help us gain insight, but they may or may not be good depending on the circumstances

If you only had a height/pressure field, what could you say about the winds?

- Direction usually follows height lines
- Speed of wind is related to height gradient (faster when lines are closer together)

- WHY?

How do we get the geostrophic wind?

Assume inertial accelerations are small in magnitude compared to the Coriolis acceleration and pressure gradient force, so…

$D\mathbf{V}/Dt \sim 0$ ($D\mathbf{V}/Dt$ is the inertial acc.)

We could take the laws of motion and then get

$0 = fv - 1/\rho\, \partial p/\partial x$ or $0 = fv - \partial \varphi/\partial x$ AND

$0 = -fu - 1/\rho\, \partial p/\partial y$ or $0 = -fu - \partial \varphi/\partial y$

Solve for u and v

If we solve for u and v in these 2 equations and call them u_g and v_g, we get equations for the Geostrophic Wind:

$u_{g\,=}$ – $1/f\rho\ \partial p/\ \partial y$ or $u_{g\,=}$ – $1/f\ \partial \varphi/\ \partial y$

$v_{g\,=}$ $1/f\rho\ \partial p/\ \partial x$ or $v_{g\,=}$ – $1/f\ \partial \varphi/\ \partial x$

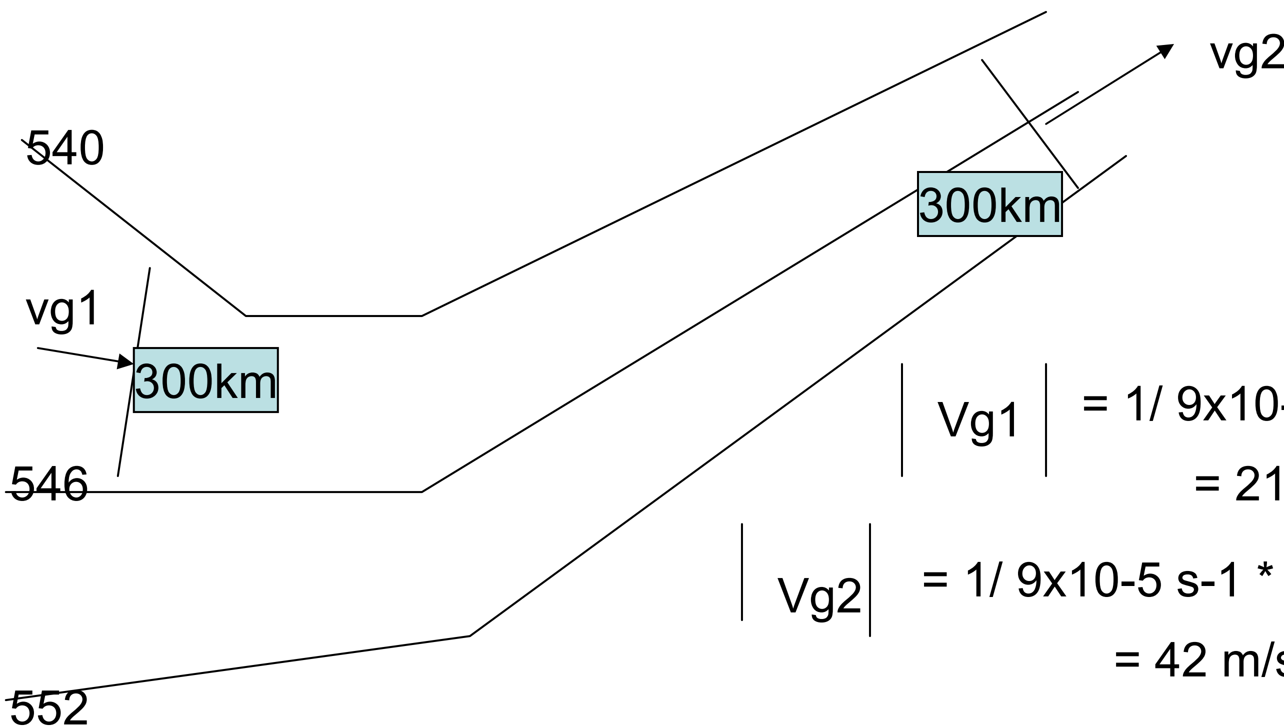
(NOTE: Need to know both forms since our surface maps show pressure, but other levels show geopotential height

Vector form:

- $\mathbf{V}_{g\,=}\quad 1/f\rho\ k \times \nabla p$ or $\mathbf{V}_{g\,=}\quad 1/f\ k \times \nabla \varphi$
- Cross-product means the geostrophic wind blows parallel to isobars/height lines with lower values of p or height to the left in the Northern Hemisphere
- Also, speed is inversely proportional to spacing between the p or φ lines (perpendicular to $\bar{\nabla}\varphi$)

Example to compute $\mathbf{V}_g$

Assume around 40 N
where f is $9\times10^{-5}\ s^{-1}$



$$|V_{g1}| = 1/\ 9x10\text{-}5\ s\text{-}1 * g\ (60\ m)/3x10^5\ m$$

$$= 21\ m/s$$

$$|V_{g2}| = 1/\ 9x10\text{-}5\ s\text{-}1 * g\ (120\ m)/3x10^5\ m$$

$$= 42\ m/s$$

Direction is parallel to height lines or from 300^o for Vg1 and 250^o for Vg2

Alternate approach

- Could also solve this by computing u_{g1} and v_{g1} using earlier equations for components, and then using
- $Tan^{-1}\ u_g/v_g$ to determine angle for wind direction. Be careful, though, since we need to think in terms of meteorological degrees, which are different from math…

Ageostrophic wind?

- Any wind can be thought of as being part geostrophic and part ageostrophic
- $\mathbf{V} = \mathbf{V}_g + \mathbf{V}_a$

- Ageostrophic wind is useful for forecasting, as it is the part of the wind that gives divergence/convergence, and crosses isobars

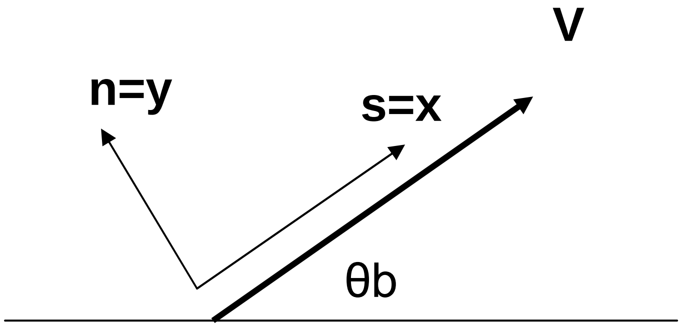
Is there a better approximation?

- Geostrophic approximation assumed no acceleration of wind, so it won't work well if there is curvature in height/pressure lines.
- Gradient wind takes curvature into account (centripetal acceleration), so it considers one extra thing beyond geostrophic wind, but still ignores friction

Gradient Wind

- More complicated to compute, and usually derived in natural coordinate system

Natural Coordinate system



Gradient wind (cont)

Dv/Dt

Gradient Wind (cont)

$D\mathbf{V}/Dt = D/Dt\ (V\mathbf{s}) = V\ D\mathbf{s}/Dt + \mathbf{s}\ DV/Dt$

But $D\mathbf{s}/Dt$ can be shown to be $= V/R_t\ \mathbf{n}$ where R_t is the radius of trajectory

So $D\mathbf{V}/Dt = V^2/R_t\ \mathbf{n} + \mathbf{s}\ DV/Dt$

Recall that Coriolis force always act to right of wind, or in **n** direction

Pressure gradient force can act in both **s** and **n** directions

So we can write our 2 component equations as…

- S component $DV/Dt = 1/\rho\ \partial p/ \partial s$
- N component $V^2/R_t = -fV - 1/\rho\ \partial p/ \partial n$
- In other words, speed changes (DV/Dt) are only due to the pressure gradient in the direction air is moving ($1/\rho\ \partial p/ \partial s$)
- And direction changes (**n** direction) are caused by the Coriolis force ($-fV$) and pressure gradient in n direction ($1/\rho\ \partial p/ \partial n$)
- V^2/R_t is the centripetal acceleration

If flow is balanced…

- DV/Dt =0 and we only have to worry about our n component equation
- $0 = -fV - 1/\rho\ \partial p/\ \partial n - V^2/R_t$
- Solve for V that gives this balance – it is called the gradient wind.
- How do we solve that equation for V? (use Quadratic Formula)

Formula

- $V = -fR_t/2 \pm (f^2R_t^2/4 - R\partial\varphi/\partial n)^{1/2}$
- Only consider answers that are real (don't consider negative numbers since V is a speed)
- Direction follows height lines just like geostrophic wind, so just approximate direction using map with contours

Interesting facts about gradient wind

- Gradient wind balance can be achieved for both cyclonic and anticyclonic flow around a Low, but only for anticyclonic flow around a high.

- Proof: $V = \dfrac{-f + (f^2 - 4/\rho R_t \, \partial\varphi / \partial n)^{1/2}}{2/R_t}$

And thus V is only real if $f^2 - 4/\rho R_t \, \partial\varphi / \partial n > 0$

- For cyclonic flow around a low, Rt>0 and dp/dn is < 0 so dp/dn can be of any size
- For anticyclonic flow, $R_t<0$ and dp/dn < 0, so $dp/dn < -R_t \rho f^2/4$
- This means you won't see a tight pressure gradient around a high, only around lows
- "air can't make the curves around a high at a fast speed, so it would diverge away from the high, weakening the high until the gradient dropped enough to lower the speeds"

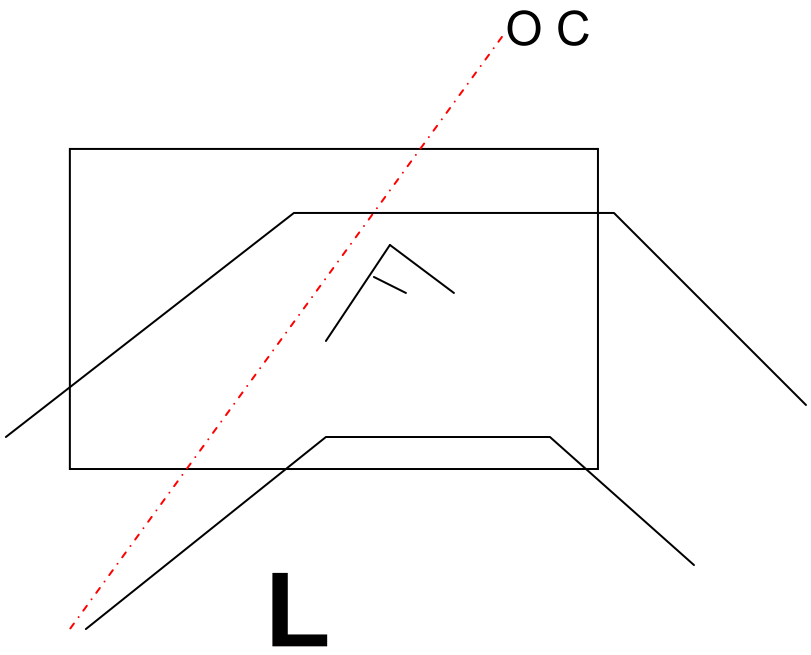
Isallobaric wind

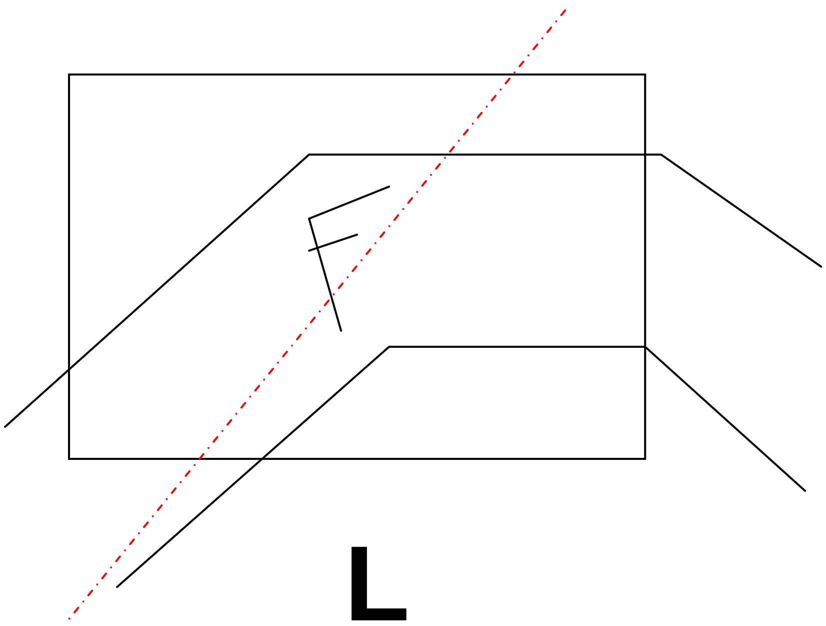
- By manipulating the equations, we can find the part of the ageostrophic wind due to the gradient of the height tendency or pressure tendency. This portion is called the Isallobaric or Isallohypsic wind. (Other parts are related to advection of the wind by itself, which can be large near jet streaks).
- The isallobaric wind blows down the pressure or height gradient toward the fastest pressure falls. It can be large for rapidly deepening cyclones or building anticyclones

Isallobaric wind

- It is an important feature to consider when wind direction will be important. Examples of this include in regions where orography influences weather (such as eastern Colorado where a NE wind would be upslope leading to lift and precip, but a NW wind downslope with drying), in a rain/snow situation in a place like Iowa where a N wind could bring down colder air, but NE would not, or in severe weather events where the isallobaric wind could create a more "backed" surface wind and more wind shear

IA example





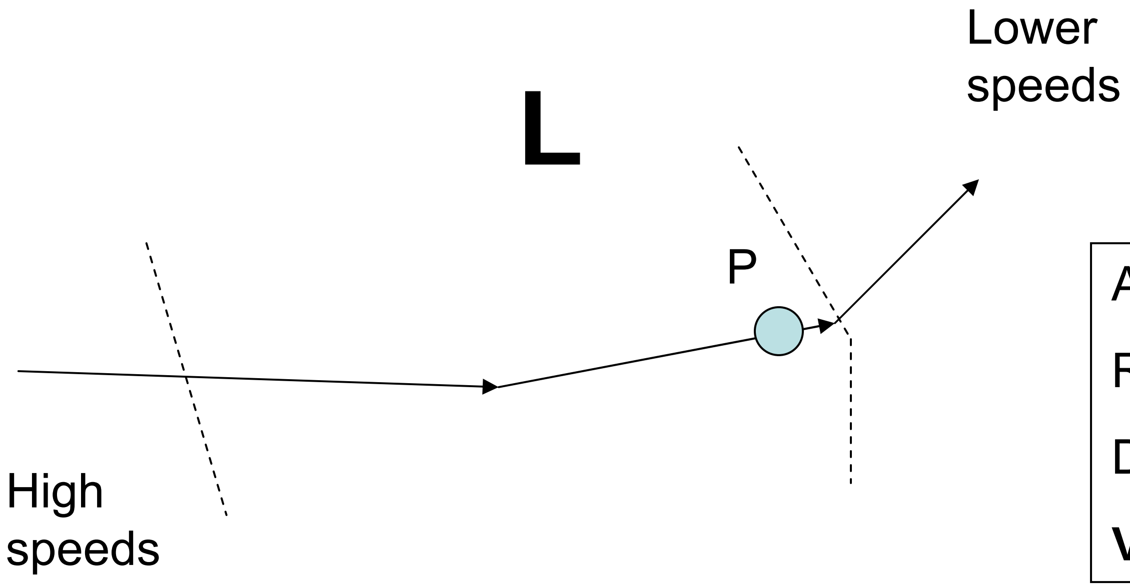
Normal cyclone has winds crossing isobars a small amount, and wind isn't really advecting colder temperatures in, so Ames might stay a cold rain

In a rapidly deepening cyclone, winds would cross the isobars more, and could advect down colder air, changing rain to snow

Force balances

- These allow us to picture in detail why air is moving the way it is
- If there are downstream changes in wind speed, then **s**DV/Dt ≠ 0
- Note that we usually care about full $\mathbf{V}_a$ which should be computed using $\mathbf{V}-\mathbf{V}_g$

Consider this example…



At point P, we know

Rt > 0

DV/Dt < 0

V, PGF

In general, we know

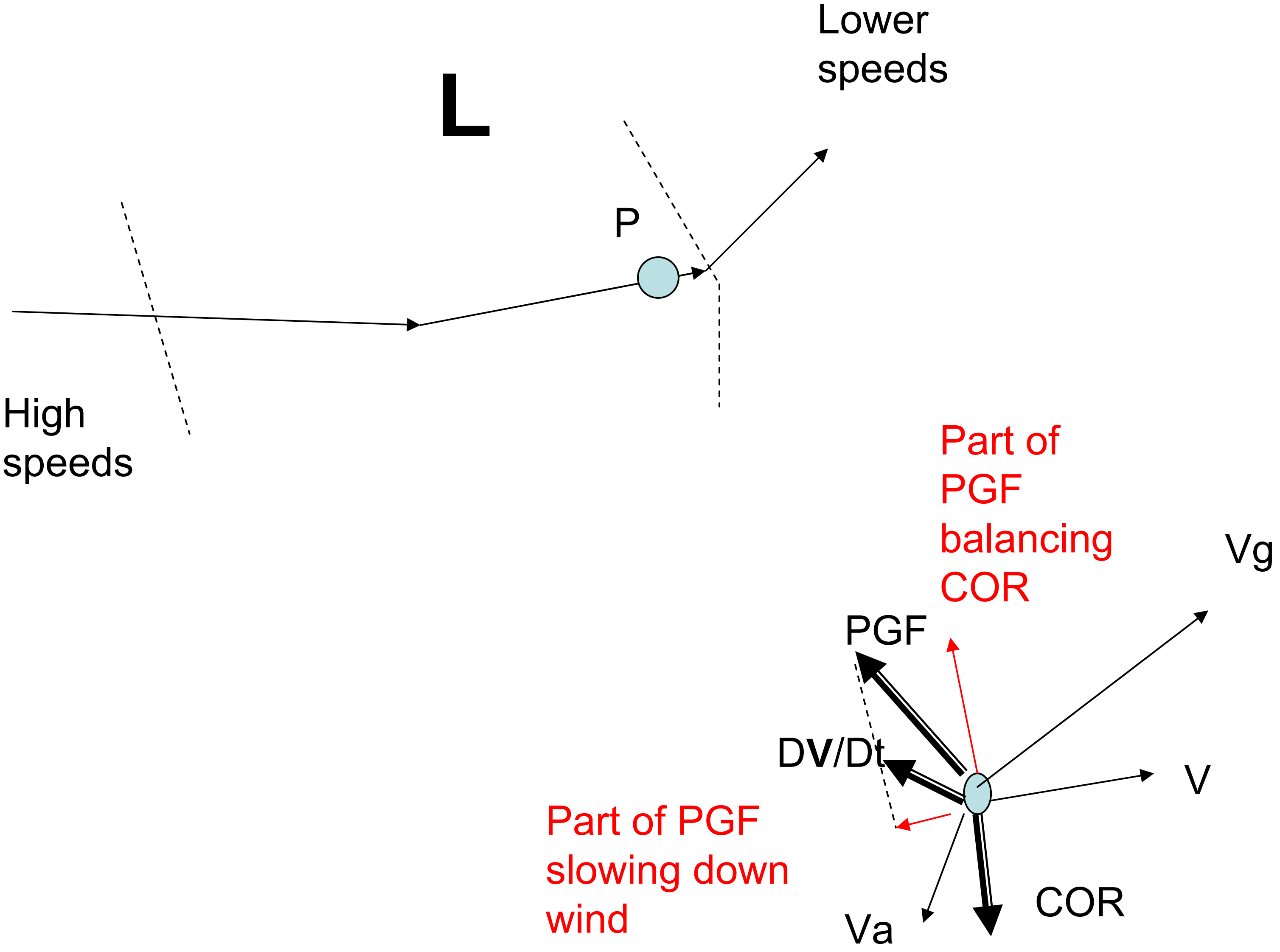
$\mathbf{V}_g$ | PGF (and we can compute Vg magnitude)

Cor | **V**

$\mathbf{V}_a = \mathbf{V} - \mathbf{V}_g$

D**V**/Dt | $\mathbf{V}_a$

SO the force balance diagram looks like…



Force balance diagrams

- When drawn correctly and to scale, one can thus see that one component of the acceleration vector is the centripetal acc and is normal to V and balances the Coriolis force, while the other, along the streamline slows down or speeds up wind.