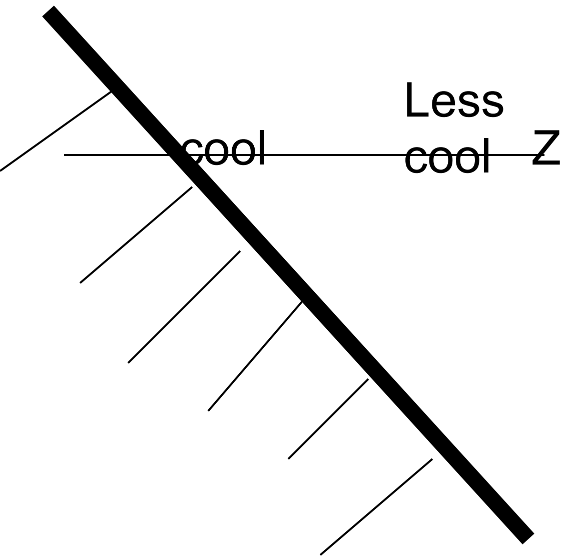
Slope and Valley winds

Another type of thermally-forced (or externally forced) mesoscale circulation → the slope wind, which is driven primarily by buoyancy (though it can be interpreted also in terms of PGF)



- Begin with an illustrative example: nocturnal slope flow, or katabatic flow (katabatic is a general term for downslope wind; the upslope counterpart is anabatic)
- Consider some height Z relative to sea level. Around sunset the land surface cools rapidly. At the same height Z but away from the terrain, the air cools relatively little. Thus, a parcel of air adjacent to the slope is negatively buoyant and will tend to accelerate downhill

Simplified solution for thermally-driven slope winds

- The following solution is similar to that developed by Prandtl (1942)
- Assume:

-- we have a 2D slope

-- time (space) scales are small enough to neglect the Coriolis force

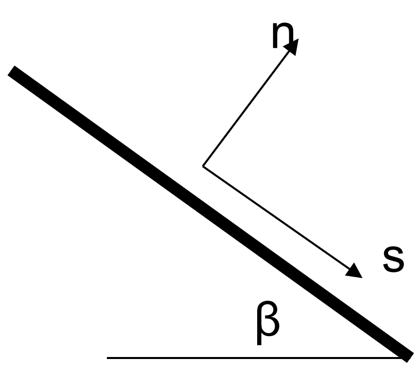
-- steady state

- We will use a rotated coordinate system (s,n) with s pointed along the slope (positive downslope) and n directed normal to the slope. The slope angle above the horizontal plane is β, and the base state potential temperature stratification is $\delta=\partial\theta_0/\partial z$ (note NOT $\partial\theta_0/\partial z$)
- A simplified governing set of equations is:

U-momentum: $\partial u/\partial t = 0 = g\theta'/\theta_{00}\sin\beta + K_m\partial^2 u/\partial n^2$

Component of buoyancy along slope

friction



- Thermodynamic equation:

$$\partial\theta' /\partial t = 0 = -u\sin\beta\delta + K_h\partial^2\theta' /\partial n^2$$

Vertical advection of base-state θ by perturbation wind

Diffusion of θ

Notice the advection term is linear because it involves the advection of the base state θ by the perturbation wind. Since $u\sin\beta=w$ where w is the cartesian vertical velocity, and $\delta=\partial\theta_0/\partial z$, this term works out to $-w\partial\theta_0/\partial z$ which is more familiar as vertical advection.

- Assume that the base state and terrain geometry are known and that we can specify K_m, K_h. Then we have 2 linear equations in two unknowns, u and θ', so we can look for a solution.
- Take $\partial^2/\partial n^2$ of the θ' eq: $0 = -\partial^2 u/\partial n^2 \sin\beta\delta + K_h \partial^4\theta'/\partial n^4$ (3)
- Solve the u eq. for $\partial^2 u/\partial n^2 = -g/K_m \theta'/\theta_{00} \sin\beta$ (4)
- And substitute into (3): 0 =

$(-g/K_m\ \theta'/\theta_{00}\sin\beta)\sin\theta\delta + K_h\partial^4\theta'/\partial n^4$ (5)

- In (5), $g/\theta_{00}\delta = g/\theta_{00}\partial\theta_0/\partial z = N_0^2$ where N is the basic state Brunt-Vaisala frequency

- Then we can rewrite (5) as:

$0 = N_0^2/K_m \sin^2\beta\theta' + K_h \partial^4\theta' /\partial n^4$ (6)

Or $\partial^4\theta' /\partial n^4 + N^2\sin^2\beta/K_m K_h\ \theta' = 0$ (7)

This is a 4th order differential equation for which the characteristic eqn is $\lambda^4 + a = 0$

The characteristic eqn can be factored into

$(\lambda+b)(\lambda-b)(\lambda+ib)(\lambda-ib)$

$= (\lambda^2-b^2)(\lambda^2+b^2)$

$= \lambda^4 - b^4$

Which corresponds to our characteristic eqn if we let $a = -b^4$ or $b= (-a)^{1/4} = -ia^{1/4}$

- Of the 4 roots, the only physical, bounded solution is

  $\theta'(n) = \theta_0' \cos(n/l)\exp(-n/l)$

Where θ_0' is the potential temperature perturbation at the ground-air interface

$l = (4K_mK_h/N^2\sin^2\beta)^{1/4}$

Substituting $\theta'(n)$ into our original equation (and omitting extensive intermediate steps), we get…

$u(n) = -g/N\,(K_m/K_h)\text{-}1/2\ \theta_0'/\theta_{00}\ \sin(n/l)\exp(-n/l)$

$= U(\theta_0')\sin(n/l)\exp(-n/l)$

Before computing numerical examples, let's look at the solution to get a general idea of the behavior:

Height variations:

*superimposed exponential decay and periodic oscillation

*the velocity has a jet-like profile: $u=0$ at $n=0$ (since $\sin(0)=0$); increases to a maximum at some height above the surface, then decreases *find height of velocity maximum by setting $\partial u/\partial n = 0$

$U(\theta_0') \, \partial/\partial n[\sin(n/l)\exp(-n/l)] = 0$

$= U(\theta_0') \partial/\partial n[\cos(n/l)\exp(-n/l)-\sin(n/l)\exp(-n/l)]$

$= U(\theta_0') \, 1/l \, \exp(-n/l) \, [\cos(n/l) - \sin(n/l)]$

Thus we see that $\partial u/\partial n=0$ when $\cos(n/l)=\sin(n/l)$, which occurs at $n/l = \pi/4, 5\pi/4, 9\pi/4, \ldots$

Given the exponential decay term, the highest value of u will be at the first value, $n=\pi/4$. Local max or min then occur at $5\pi/4, 9\pi/4, \ldots$

- Above n=π/l, the solution reverses sign, and we have upslope flow; the sign of the flow reverses again above n=2πl, and so on.
- The value of the maximum velocity is then obtained by solving for n=πl/4:
- $U_{max} = U(\theta_0') \sin(\pi/4)\exp(-\pi/4)$
$\approx 0.32\, U(\theta_0')$

- The intensity of the slope flow, $U(\theta_0')$, is linearly proportional to the intensity of surface cooling, θ_0', and inversely proportional to the Brunt-Vaisala frequency, N, (i.e. the square root of the potential temperature gradient). The slope flow also is inversely proportional to the square root of the turbulent Prandtl number ($P_{rT} = K_m/K_h$) but usually P_{rT} is close to 1 and varies only slightly (with stability).

Numerical example

- θ_0' = -5K, θ=288K, ∂θ/∂z=9.8K km-1 (isothermal) so N≈0.018s-1, N^2≈3x10-4s^{-2}
- β=5^o → sinβ = 0.09
- K_m=K_h=1 m2s^{-1} (turbulent velocity scale ≈10 cm s^{-1} and turbulent length scale of 10 m)
- Then U(θ_0') = (-9.8ms^{-2}/0.018s^{-1})(1)(5K/288K) = 9.4 ms^{-1}
- l = (4*1m^2s^{-1}*1m^2s^{-1}/3x10-4s^{-2}*$(0.09)^2$)1/4 = 36m
- The max slope flow velocity is when n=πl/4 or 28 m and equals 0.32 U(θ_0') = 3 ms^{-1}.