

Instabilities and Basic Convection



Buoyant Instability (gravity is restoring force)

- Assume a stationary incompressible fluid (like water), so that $\rho = \rho_0 + \partial\rho/\partial z z$ and also it is in hydrostatic equilibrium so we can substitute $\partial p/\partial z = -\rho g$

Assume small horizontal tube of fluid with $\rho = \rho_0$ at $z=0$

Displace upward to height z

- We can get our equation to look like:

$$\rho_0 dw/dt = \partial p / \partial z - g \rho_0 = g \partial \rho / \partial z z$$

$$(1) w = dz/dt \rightarrow d^2z/dt^2 = (g \partial \rho / \rho_0 \partial z) z$$

If initial displacement is to $z=h$, the solution to (1) is

$$z = h \cos Nt \text{ with } N^2 = -g / \rho_0 \partial \rho / \partial z$$

In a compressible atmosphere (unsaturated) $\rightarrow \theta$ is conserved and not ρ , so $N = (g / \theta_0 \partial \theta / \partial z)^{1/2}$

- Inertial/Symmetric Stability/Instability

→ rotation has a stabilizing effect

Consider 1) mean zonal flow

2) balanced geostrophically with
N-S gradient of pressure

Then $\partial p / \partial y = -\rho f u$ (from defn of u_g)

$M = u - f y$ is defined as local approx. of
absolute angular momentum.

M is conserved in tangent plane equations
of motion since...

$dM/dt = d/dt(u-fy) = du/dt - fv = 0$ (friction is neglected and f assumed locally constant)

So tube displacement increases its u by $f\Delta y$, but ends up supergeostrophic, so it returns south.

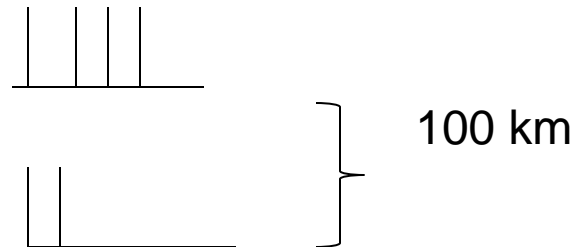
$$dv/dt = f(u_g - u)$$

unless u_g also increases \rightarrow acc. Is southward. Thus, if environment mean northerly shear $\partial u / \partial y < f$, it is stable to horizontal displacements.

We generalize this to 2D for a requirement for inertial stability to be:

$$f + \partial v / \partial x - \partial u / \partial y = \zeta_{\text{abs}} > 0$$

This basically says that major anticyclonic vorticity is inertially unstable



Now, consider horizontally and geostrophically balanced mean flow with vertical shear present. Here, buoyancy and rotation combine to create a new buoyant-inertial instability

Consider mean flow and potential temp. field satisfying thermal wind equation:

$$f\partial u/\partial z = -g/\theta_0 \partial\theta/\partial y$$

We have instability if $\partial z/\partial y|_{\theta} > \partial z/\partial y|_M$

(stability if instead $<$) This means if θ lines slope more steeply than M lines in 2D vert. cross-section, we have symmetric instability

- We can use the chain rule...

$$\partial z / \partial y |_{\theta} = -\partial \theta / \partial y / \partial \theta / \partial z \quad \text{and}$$

$$\partial z / \partial y |_{M} = -\partial M / \partial y / \partial M / \partial z$$

So $N^2 / (\partial u / \partial z)^2 (1 - \partial u / \partial y / f) > 1$ for stability

We define $N^2 / (\partial u / \partial z)^2$ to be the Richardson number.

High Ri number = low vertical shear and large stability

If we add moisture effects and use θ_e and not θ , we have conditional symmetric instability – name due to on a baroclinic circular vortex, the motion fields are zonally symmetric

- Kelvin-Helmholtz instability – vertical shear is source of instability leading to turbulence

No simple mechanistic model to predict this

If shear is unusually strong and the stable stratification weak, we can get KH waves

To evaluate likelihood, consider two parcels (1) and (2) where (1) is higher up with speed $u + \Delta u$ and (2) is lower down with speed u .

Assume they mix and end up with same velocity afterwards to get max. reduction in kinetic energy (KE)

$$\Delta PE \rightarrow g \partial \rho / \partial z, \partial \rho = -(\partial \rho / \partial z) \partial z$$

Energy source is the mean shear $\partial u / \partial z$

Reduction in KE if both have velocity after mixing of $(u + \Delta u) / 2$

$$\Delta KE = 1/2 \rho [u^2 + (u + \Delta u)^2 - 2\{(u + \Delta u) / 2\}^2] = 1/4 \rho (\Delta u)^2$$

For stability, $\rho (\Delta u)^2 / 4 < g \partial \rho / \partial z$ (energy is released)

Thus, $-(g / \rho)(\partial \rho / \partial z) / (\partial u / \partial z)^2 > 1/4$ for stability

Roughly $10/1$ ($.1/1000$) = .001 in numerator

So shear of over 10 m/s per km needed

Quick Review of basics for convection

- e =vapor pressure
- e_s = saturation vapor pressure – a function of temperature only. Increases approximately exponentially with temp.

- Teten's formula:

$$e_s(T_c) = 6.11 \text{ mb} \cdot 10^{7.5T_c/T_c + 237.3}$$

- Wexler's improvement to Teten's formula:

$$e_s(T_c) = 6.112 \text{ mb} \cdot \exp(17.67T_c/T_c + 243.5)$$

This is accurate to 0.1% from $-30^\circ\text{C} < T_c < 35^\circ\text{C}$

Vapor density

- From ideal gas law

$$e = \rho_v R_v T \rightarrow \rho_v = e / R_v T$$

Where R_v is the gas constant for water vapor; $R_v \approx 462 \text{ J kg}^{-1} \text{ K}^{-1}$

Mixing ratio $r = \rho_v / \rho_d$

Specific humidity $q = \rho_v / (\rho_v + \rho_d)$

so $r = q / (1 - q)$

Ballpark numbers... $q, r \approx 20 \text{ g/kg}$, $e \approx 20 \text{ mb}$

Convection and convective systems

- Parcel theory – best seen using skew-T

$$\theta_e = \theta \exp(Lq_v/c_p T), \quad L=f(T); \quad c_p=f(\text{mol. wt.})$$

LCL (lifting condensation level)

LFC (level of free convection)

ETL (equilibrium level)

All of these 3 items are properties of specific parcels. Different layers of the sounding will have different values of these

More....

- “overshooting top”
- Positive area, negative area \rightarrow energy
- Parcel rises due to buoyancy. The buoyant acceleration can be defined as

$$dw/dt = g (\rho_{env} - \rho) / \rho \approx (\theta - \theta_{env}) / \theta_{env} = g(T - T_{env}) / T_{env}$$

The work done by the rising parcel on the environment (per unit mass) is force acting through a distance.

$$W = \int_{z_1}^{z_2} (g (T - T_{\text{env}}) / T_{\text{env}}) dz$$

If we consider the layer through which the parcel is positively buoyant, the ability of a parcel to do the work of rising from the bottom to the top of this layer is referred to as the potential buoyant energy (PBE) of the parcel (or CAPE)

$$\text{PBE} = \int_{\text{LFC}}^{\text{ETL}} (g (T - T_{\text{env}}) / T_{\text{env}}) dz$$

The PBE may not always be released (i.e. converted to the kinetic energy of rising)

It is first necessary for the parcel to reach its LFC. Once this happens, the PBE becomes available buoyant energy.

The parcel usually will be negatively buoyant in some layer below the LFC. This negative buoyant energy is defined analogously to the PBE;

$$\text{NBE} = \int_{z_1}^{\text{LFC}} (g (T - T_{\text{env}}) / T_{\text{env}}) dz \quad (\text{also CIN})$$

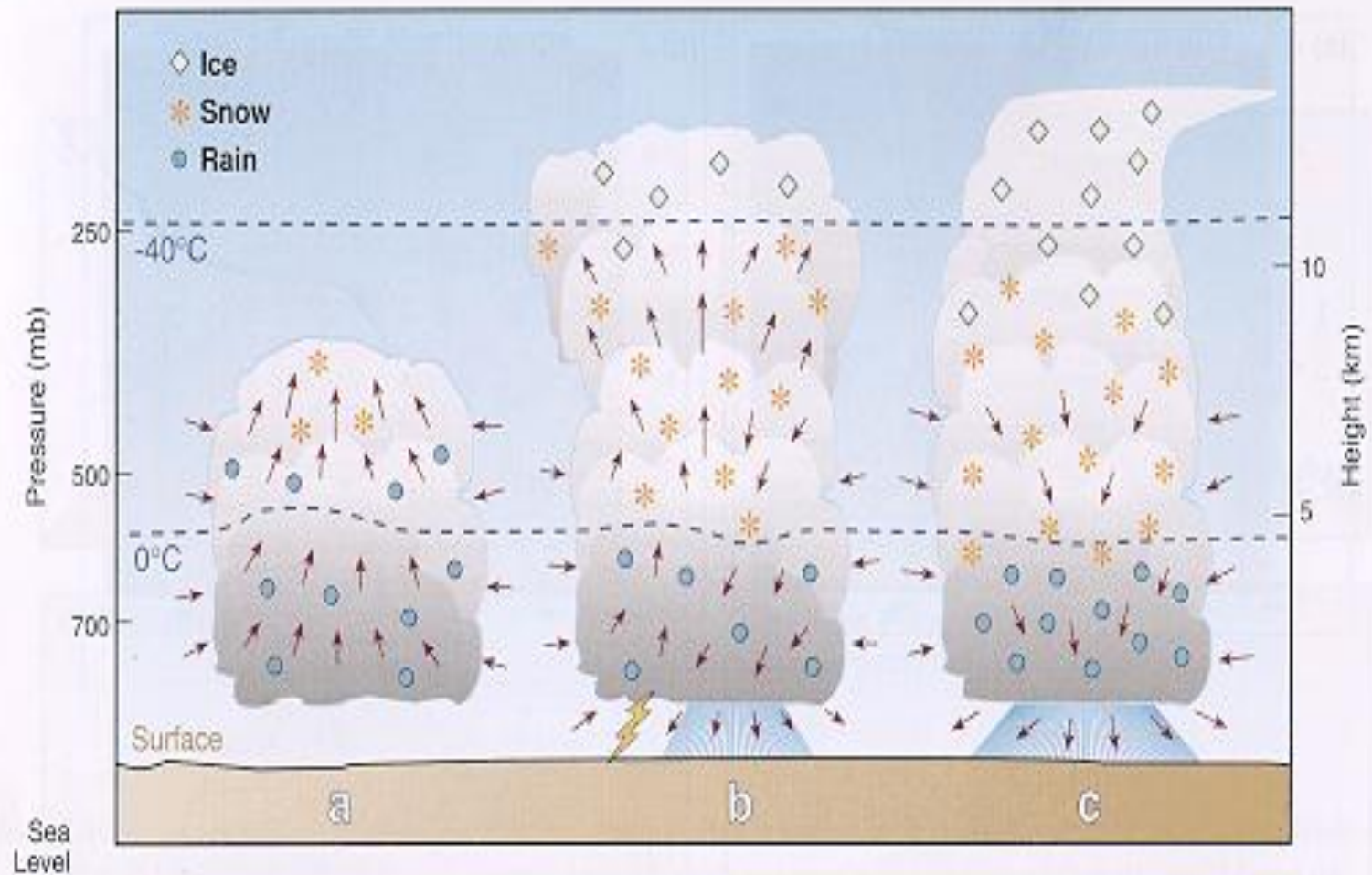
Overcoming NBE

- The NBE can be overcome in several ways. In general these can be classified as:
 - 1) erosion of the layer of NBE due to heating from below (growth of the mixed layer)
 - 2) Some type of lifting mechanism, e.g.,
 - a) frontal lifting
 - b) flow over terrain
 - c) gravity-wave osc.

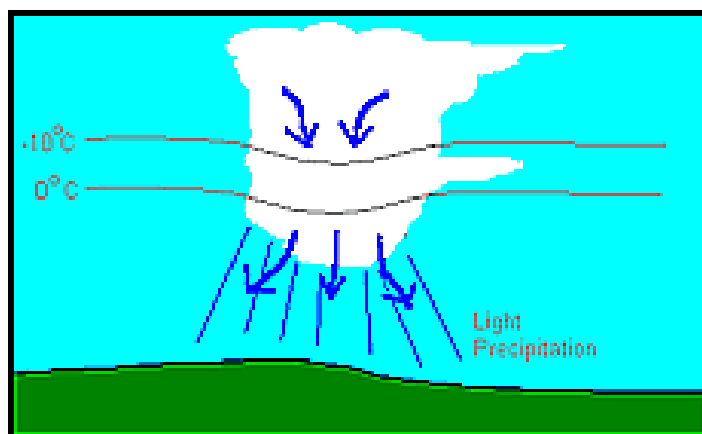
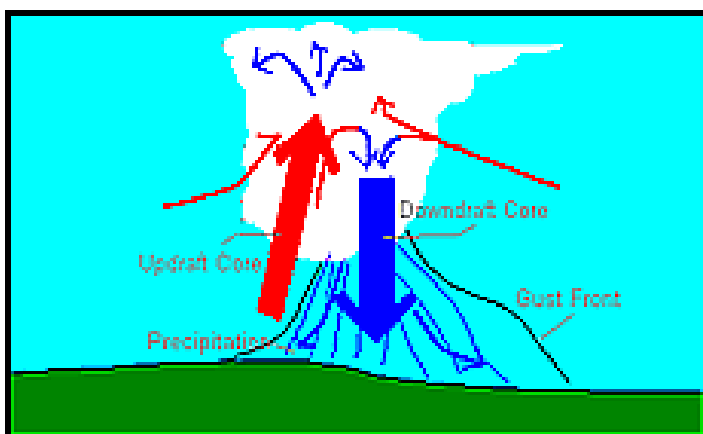
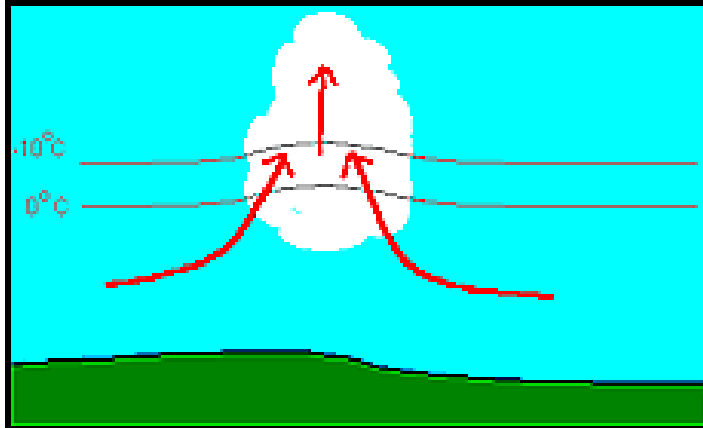
Storm Type

- Single-cell storms – usually are short-lived, ≈ 30 min. This is primarily because they occur in environments with little or no shear.
- Typically several km in horizontal extent (up to ≈ 10 km). The accumulated rainfall is usually small due to the short lifetime although the rain rate may briefly be heavy. On ave, about 20% of the condensed vapor falls as rain.

- Latent heat release drives the ascent – vertical motion increases as long as parcel Temp > environment Temp. Entrainment of dry air leads to some evaporation of cloud water/rain, which cools the parcel, and detracts some from upward motion.



Thunderstorm life cycle (from Byers and Braham 1949) – adapted by State Climate Office of North Carolina



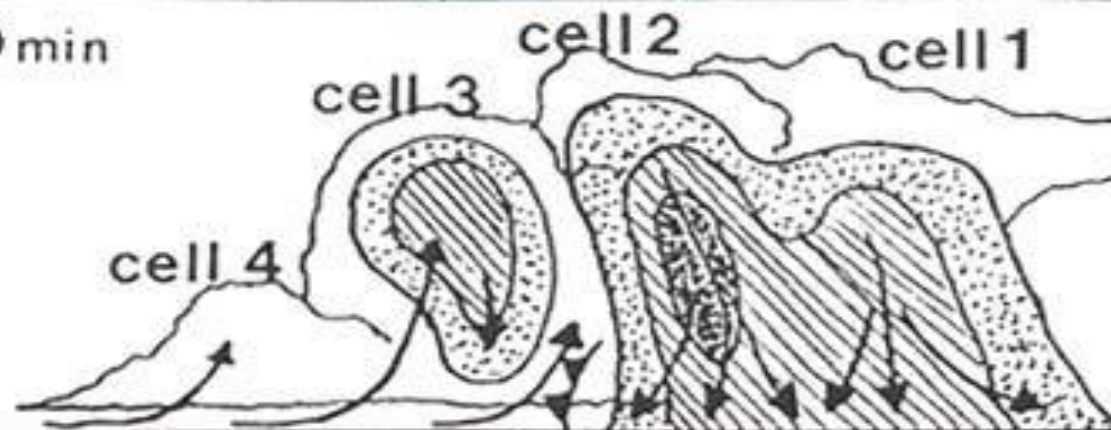
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Multicell storms

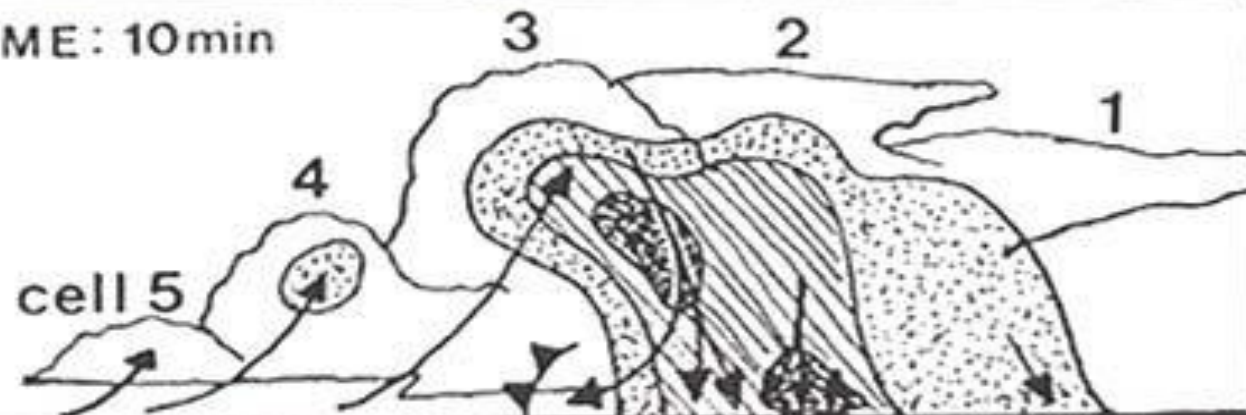
- These are composed of a collection of individual convective elements. Perhaps the best known is the squall line.
- Squall lines tend to form in environments with moderate speed shear though the directional shear is often small. This shear plays a very important role in allowing the storm to sustain itself.

- Downdraft is offset from updraft
- Storm-relative inflow occurs at low levels
- Also note that the role of the outflow is providing convergence/lifting to create new cells. The individual cells do not necessarily have very long life times, but the system can last for hours due to continual development of new cells.

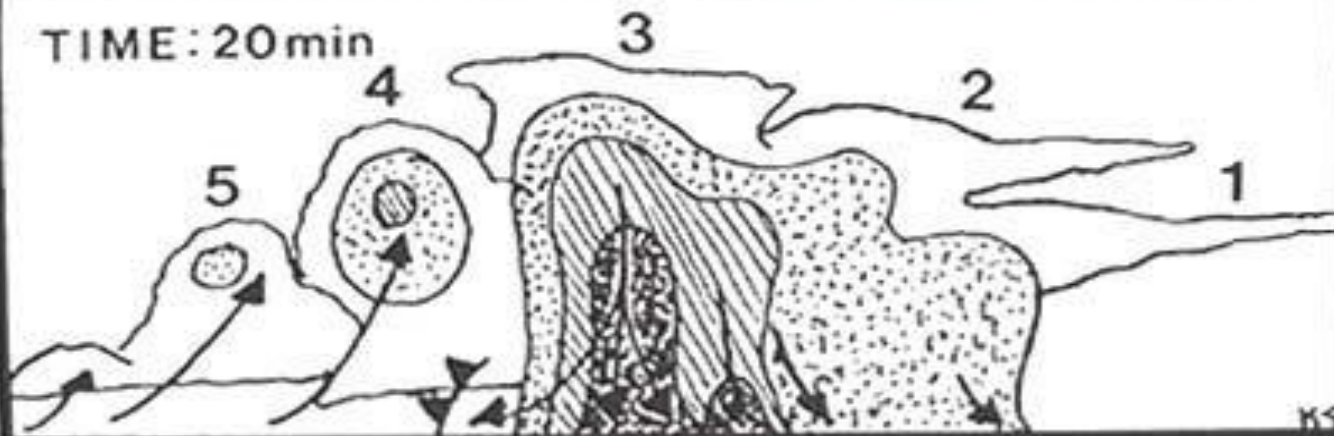
TIME: 0 min



TIME: 10 min



TIME: 20 min



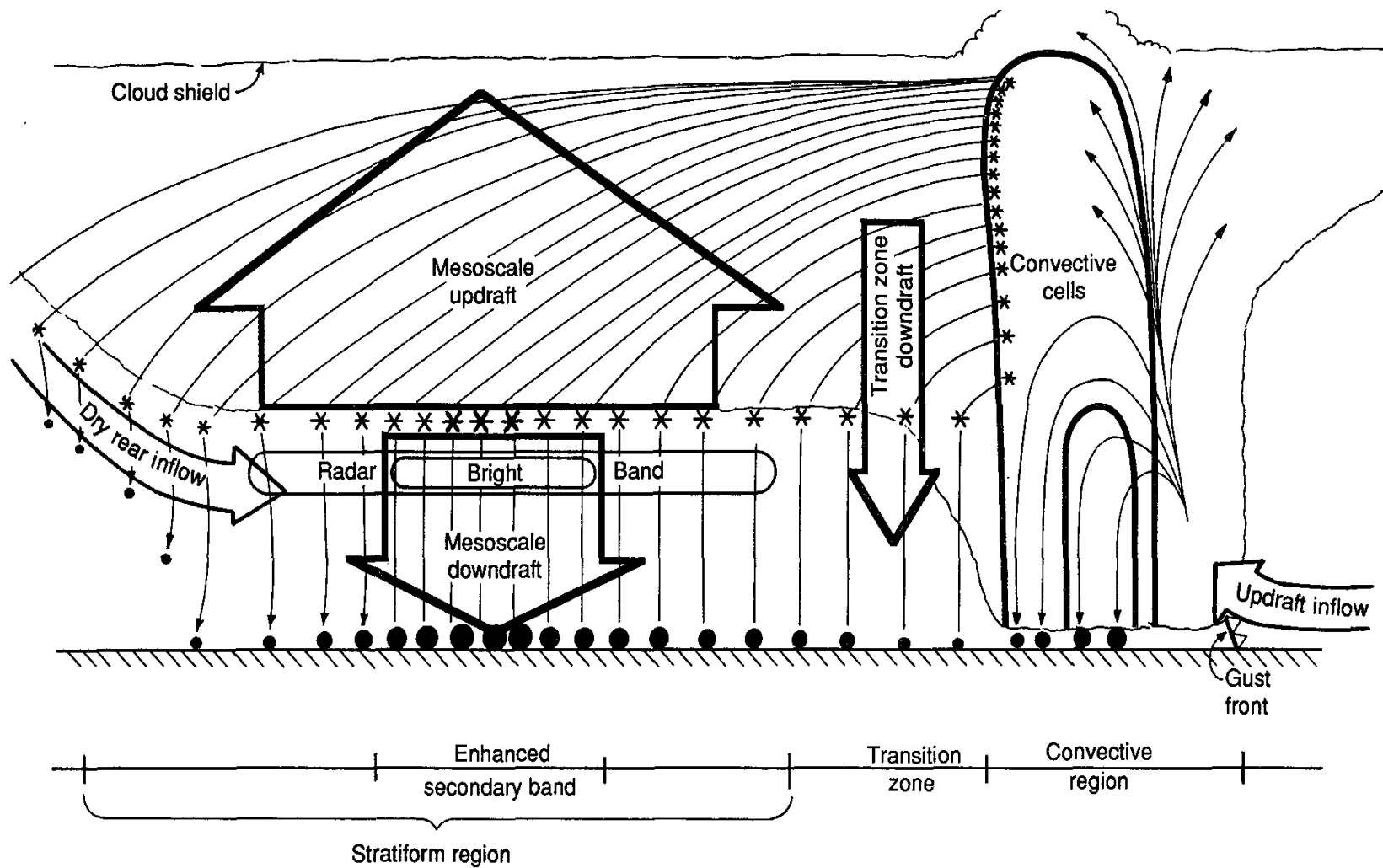


FIG. 18. Conceptual model of the two-dimensional hydrometeor trajectories through the stratiform region of a squall line with trailing stratiform precipitation. See text for further explanation.

Different types of multicells

- Squall lines
- Derecho
- Bow Echo
- Nonlinear events

These are all called MCC/MCS

Strong Echo
Tail



A

DOWNBURST



B

Cyclonic



C

Anticyclonic

Rotating Head



D

Head



E

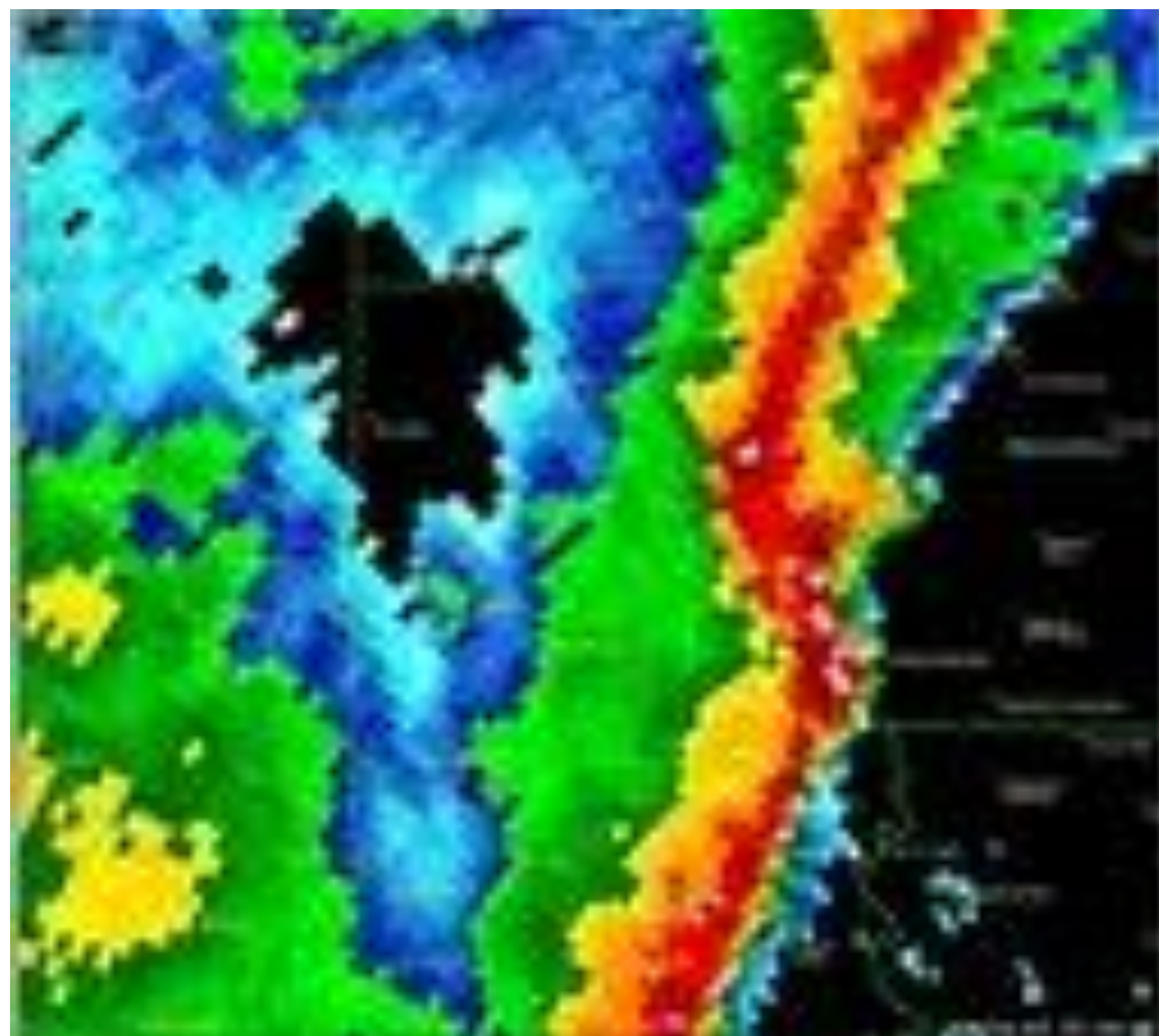
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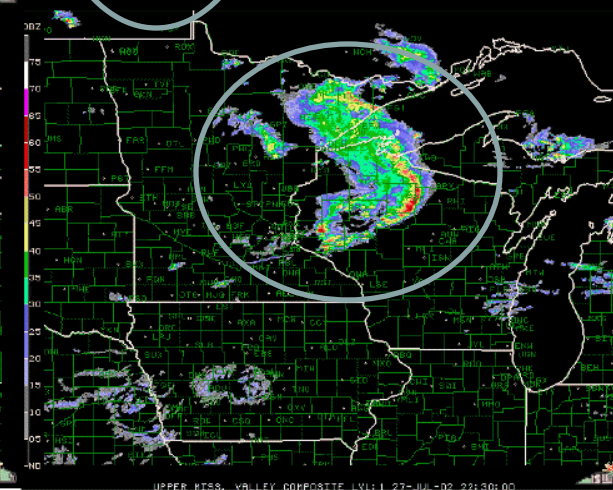
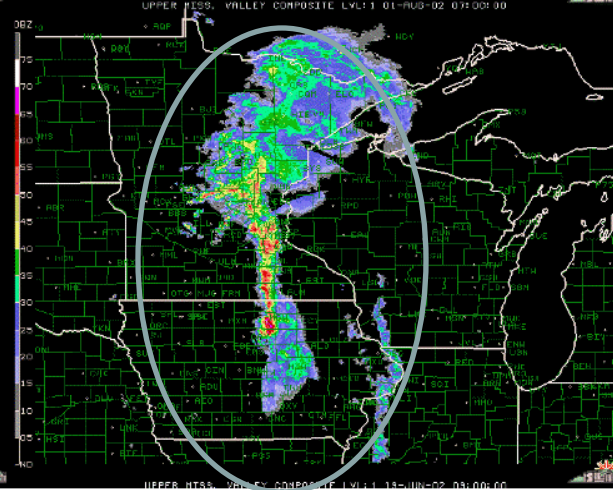
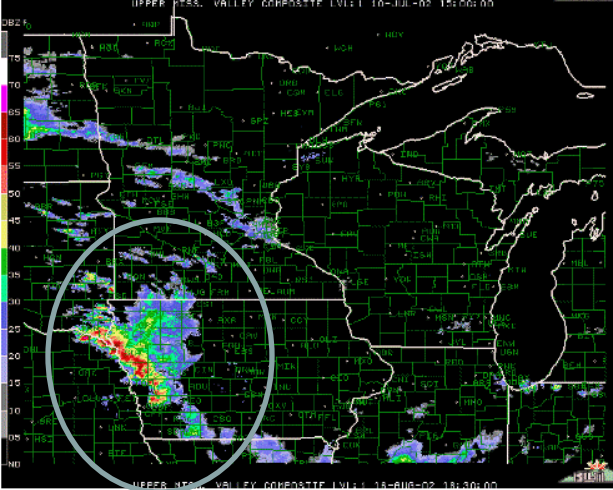
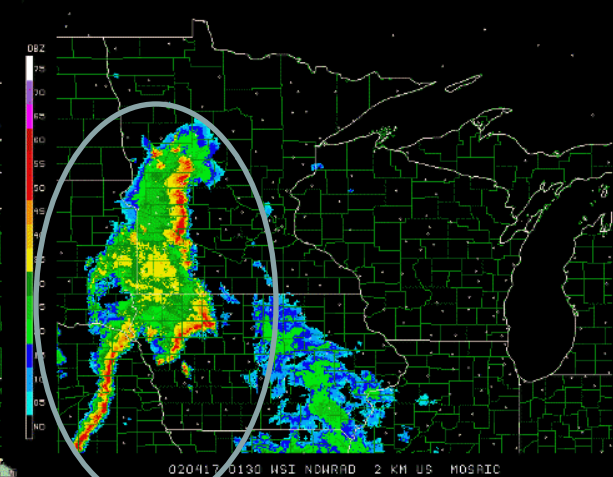
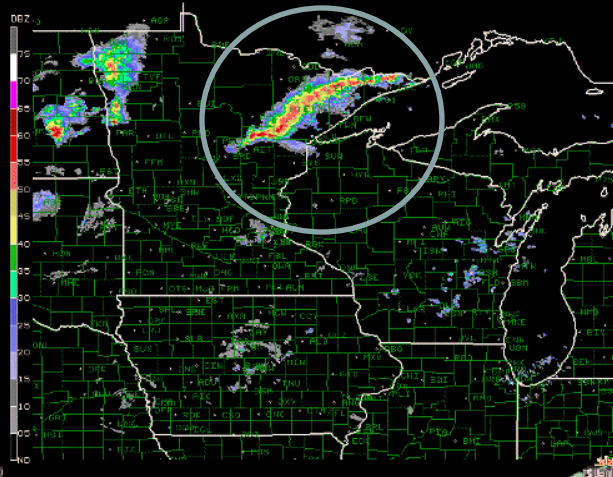
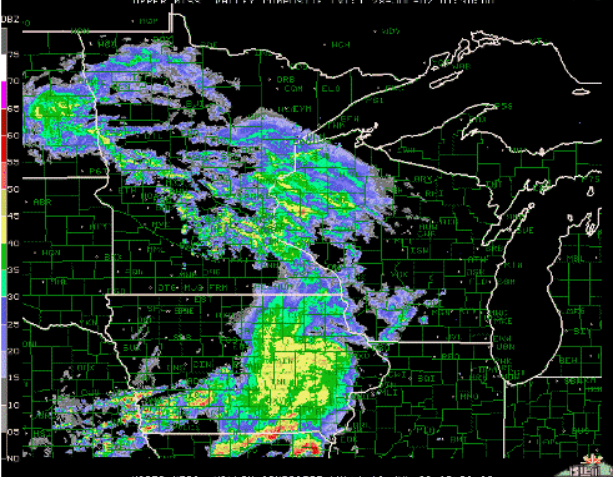
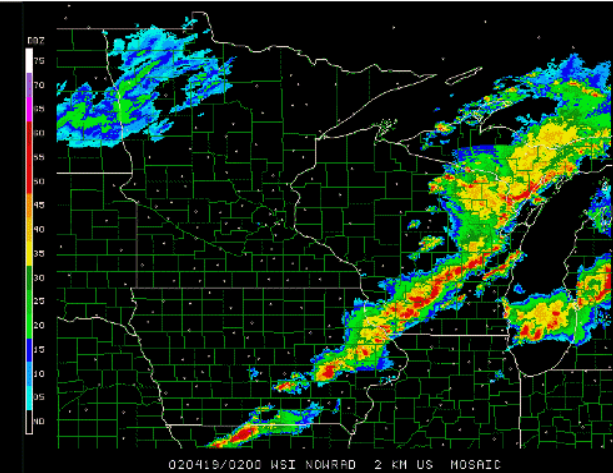
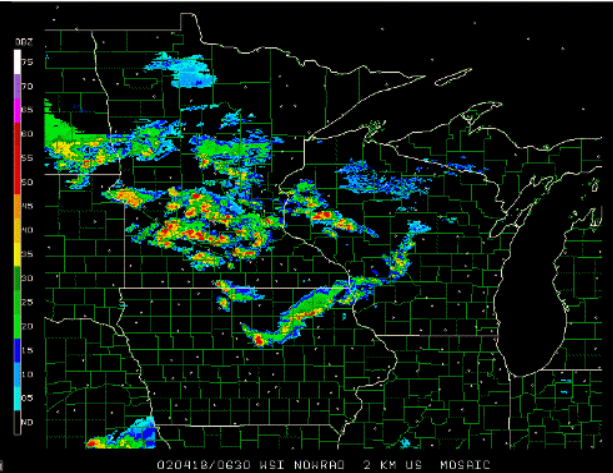
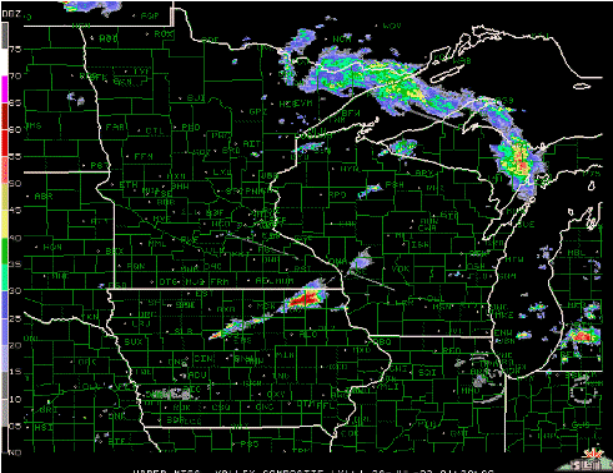
Old Tail

New Tail

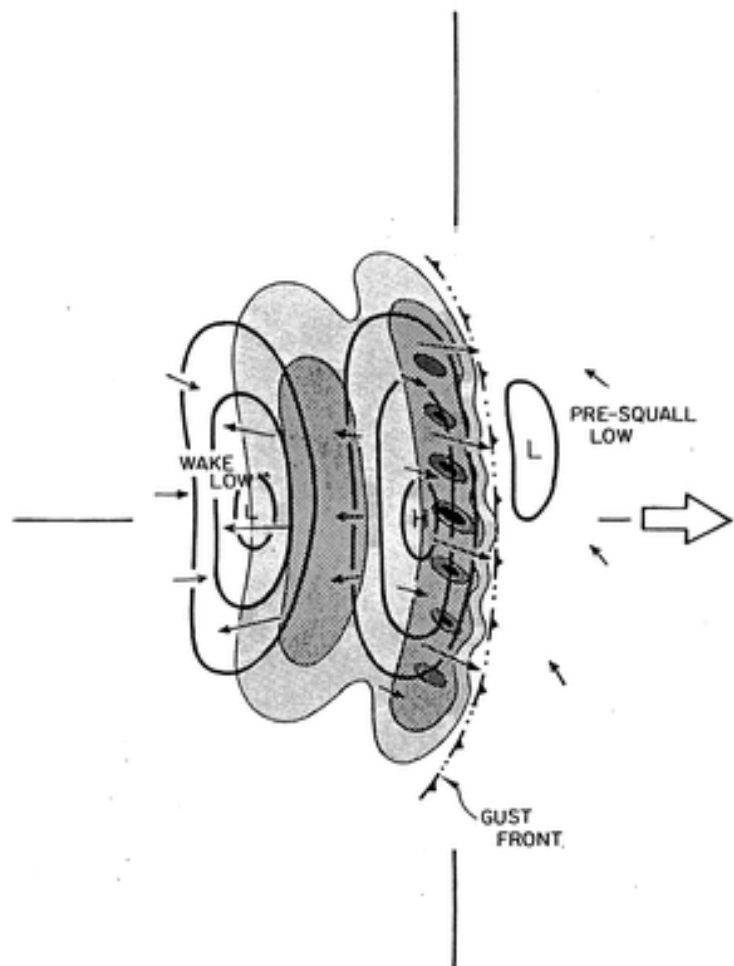
Squall lines

- Several different types of squall lines based on various factors:
 - a) trailing, leading, line-parallel stratiform rain
 - b) broken areal, broken line, embedded areal, backbuilding formation mechanisms

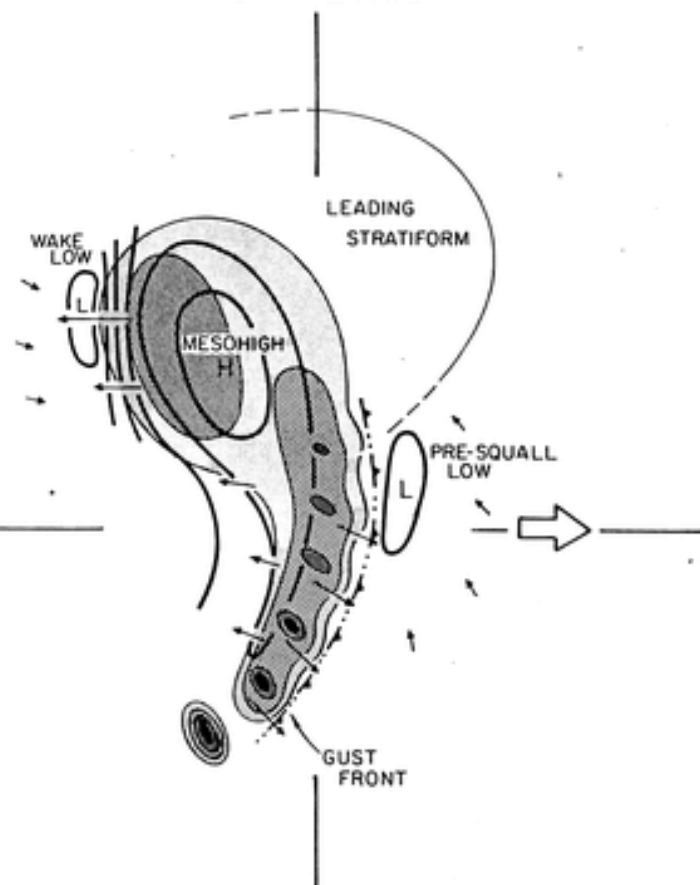




SYMMETRIC



ASYMMETRIC



CLASSIFICATION OF SQUALL-LINE DEVELOPMENT

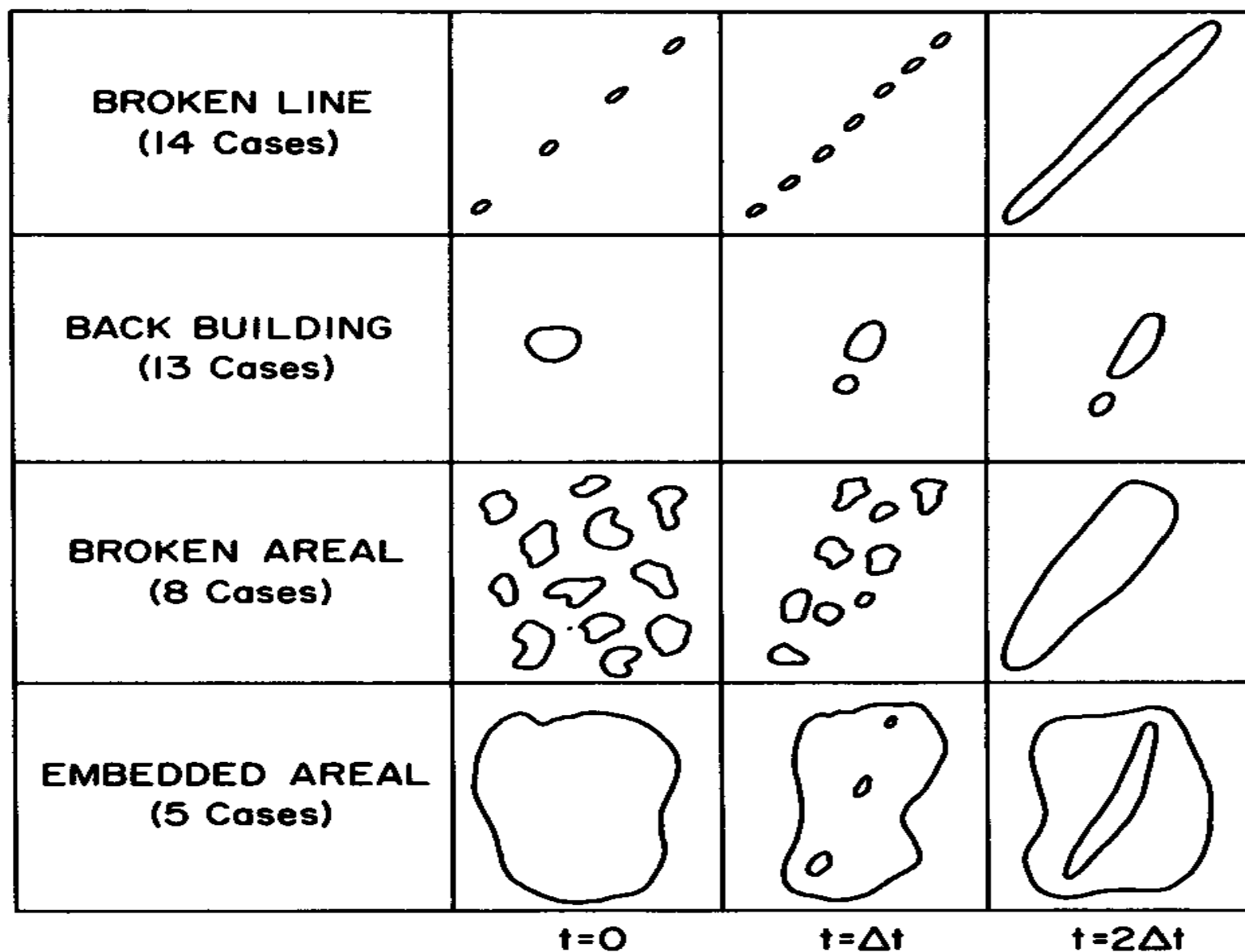


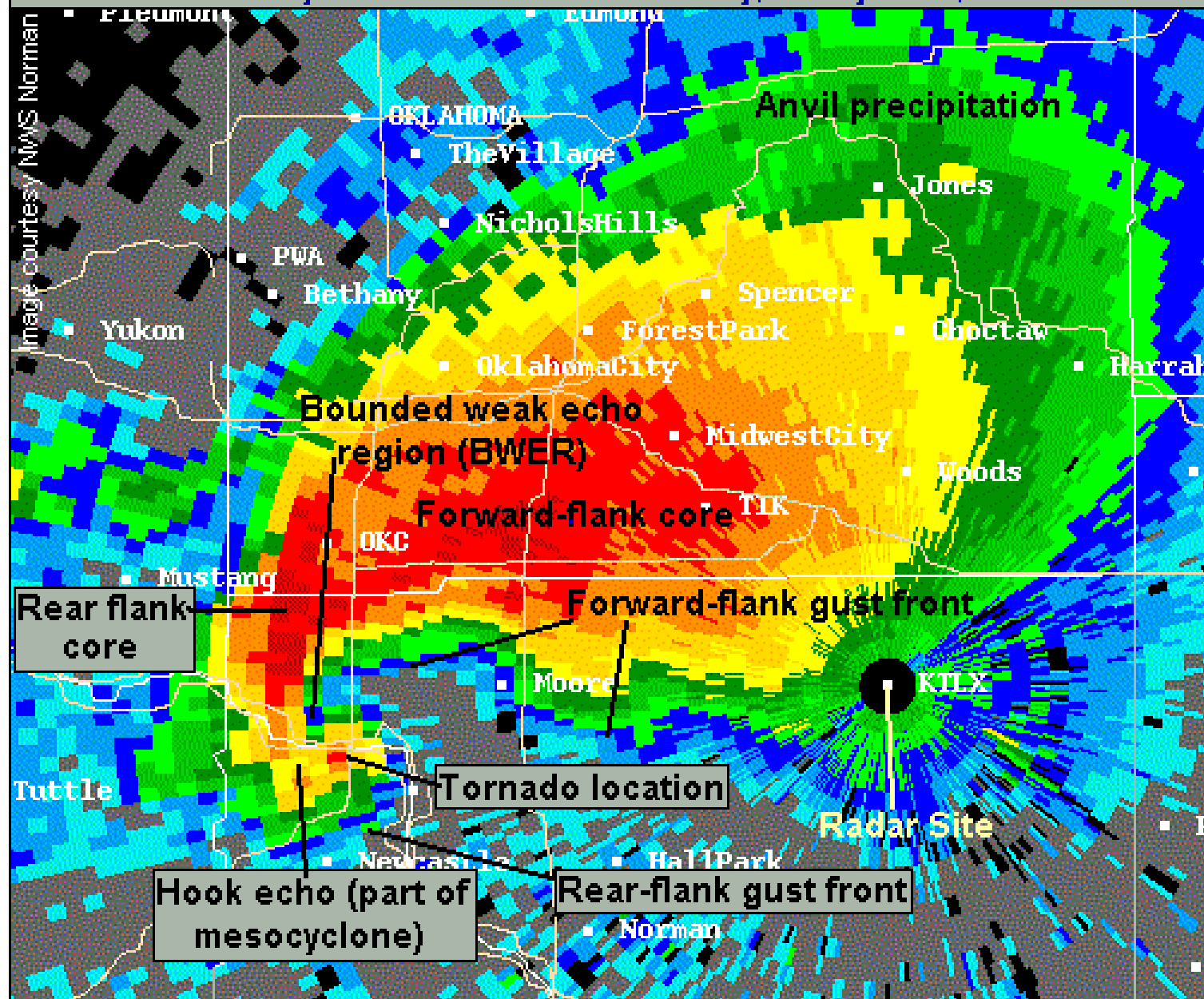
FIG. 1. Idealized depiction of squall-line formation.

Supercells

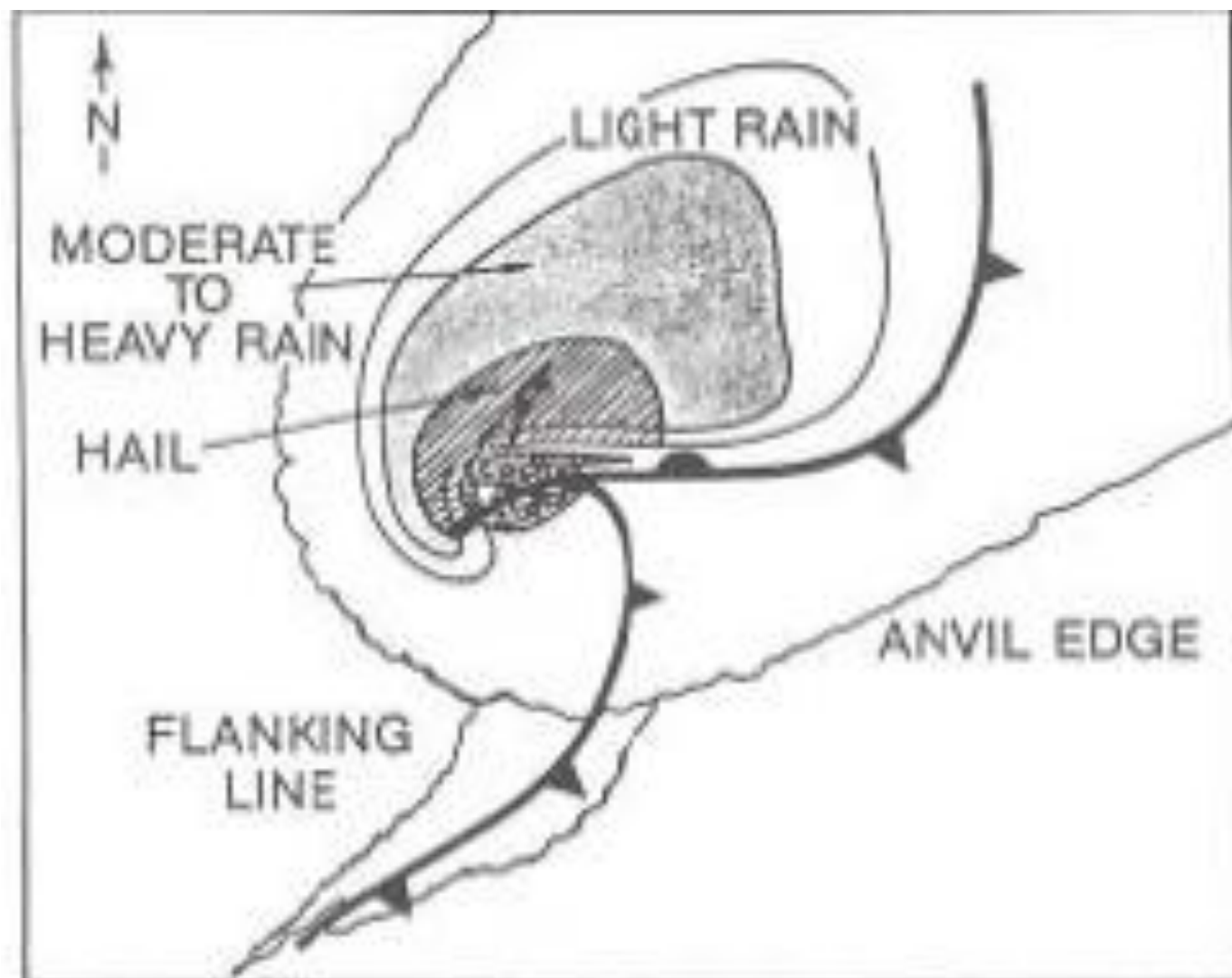
- These are also composed of smaller convective elements but are very highly organized so that in effect the storm behaves as one very large convective cell. In supercells the new updrafts that form along the edges of the storm will feed the organized updraft rather than growing into distinct storm cells. Often they last for hours and travel several hundred km.

- Supercells tend to form in environments with directional shear and moderate amounts or strong speed shear. As with multicell storms, the shear has an important part in separating the updraft from the precipitation/downdraft.

Oklahoma City NEXRAD base reflectivity, 3 May 1999, 1912 CDT



**RADAR DEPICTION OF A CLASSIC SUPERCELL
CONTAINING A VIOLENT TORNADO**



ANVIL

MAMMATUS

MAIN STORM
TOWER

FLANKING LINE
TOWERS

COLLAR TAIL

RAIN-FREE BASE

WALL

PRECIPITATION

SOUTHWEST

WEST

NORTHWEST

SCHEMATIC VIEW OF A TORNADIC THUNDERSTORM

