Jet streaks and Jetogenesis

Jet streams and jet streaks

The rate of change of the geostrophic wind with height is found from the thermal wind relation,

$$\frac{\partial \vec{v}_g}{\partial p} = \frac{-R}{fp}\left(\hat{k} \times \nabla_p T\right) \qquad \begin{array}{ccc} 0 & 0 & 1' \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & 0 \\ \bar{i} & \bar{j} & k \end{array}$$

or

$$\frac{\partial \vec{v}_g}{\partial p} = \frac{-R}{fp}\left(\frac{p}{p_0}\right)^{R/c_p}\left(\hat{k} \times \nabla_p \theta\right)$$

Consider a north-south temperature gradient (so that the front is oriented-east west). Then we need consider only $\partial T/\partial y$ (or $\partial \theta/\partial y$) in the thermal wind relation;

$$\frac{\partial u}{\partial p} = \frac{R}{fp}\left(\frac{\partial T}{\partial y}\right)_p = \frac{R}{fp}\left(\frac{p}{p_0}\right)^{R/c_p}\frac{\partial \theta}{\partial y}$$

We see:

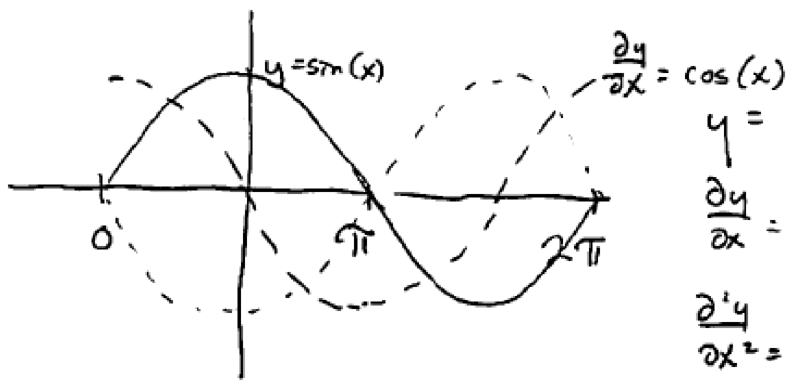
- regions of strong horizontal temperature gradients correspond to regions of strong vertical shear of the geostrophic wind.
- for $(\partial T/\partial y, \partial \theta/\partial y) < 0$ (i.e., cold to the north), we have $\partial u/\partial p < 0$ i.e., u_g increases with height (since the pressure decreases with height)

Consider an atmosphere with an east-west oriented frontal zone but no surface wind. Then to the extent that the atmosphere is in geostrophic balance, we expect the strongest winds aloft to be above the frontal zone, since that is the region where the wind has its strongest increase with height.

By the thermal wind rel.

More generally, we expect jet streams to be located above frontal zones. The wind continues to increase with height so long as the temperature gradient is maintained. Then since the entire troposphere tends to be warm in the tropics and cold at the poles, we expect that jet streams should be located near the tropopause.

A jet streak is a local maximum of wind speed within a jet stream. Recall from calculus that local maxima and minima of some variable are located where the first derivative is zero. The second derivative tells us whether the function is a maximum or minimum; specifically, the 2nd deriv. is negative for a local maximum and is positive for a local minimum. The most straightforward case is a simple sine wave:



$$y = \sin(x)$$

$$\frac{\partial y}{\partial x} = \cos(x)$$

$$\frac{\partial^2 y}{\partial x^2} = -\sin(x)$$

We see that the local maximum of $y = \sin(x)$ is indeed located where $\frac{\partial y}{\partial x} = \cos(x) = 0$, i.e. at $x = \frac{\pi}{2}$, and that $\partial^2 y / \partial x^2 = -\sin(x) = {}^{-}1$ at $x = \frac{\pi}{2}$.

Then if a <u>jet streak</u> is a <u>local</u> <u>maximum</u> of the wind speed, we expect that the jet streak is where

$$\nabla^2 u < 0$$

So we would tend to build a jet streak by making $\nabla^2 u$ more negative. Accordingly, Bluestein defines a "jetogenetical function" J as

$$J = \frac{d}{dt}\left(-\nabla^2 u\right)$$

If we expand $\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \omega\frac{\partial}{\partial p}$, we obtain

$$J = \frac{\partial}{\partial t}\left(-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial p^2}\right) + u\frac{\partial}{\partial x}\left(-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial p^2}\right) + v\frac{\partial}{\partial y}\left(-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial p^2}\right) + \omega\frac{\partial}{\partial p}\left(-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial p^2}\right)$$

$$= \frac{\partial}{\partial t}\left(-\nabla^2 u\right) + \vec{V}\cdot\nabla\left(-\nabla^2 u\right)$$

Interchanging the order of partial derivatives,

$$J = \left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial p^2}\right)\frac{\partial u}{\partial t} - \vec{V}\cdot\nabla\left(\nabla^2 u\right)$$

$$= -\nabla^2\frac{\partial u}{\partial t} - \vec{V}\cdot\nabla\left(\nabla^2 u\right)$$

We would like to somehow convert $\frac{\partial u}{\partial t}$ to $\frac{du}{dt}$ since this would allow us to incorporate the eq. of motion into J [recall the eq. of motion is cast into a Lagrangian framework; i.e., $\frac{du}{dt}$ = (forces)]. Notice:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \vec{V} \cdot \nabla u$$

Then:

$$\nabla^2 \left(\frac{du}{dt} \right) = \nabla^2 \left(\frac{\partial u}{\partial t} \right) + \nabla^2 \left(\vec{V} \cdot \nabla u \right)$$

or

$$\nabla^2 \left(\frac{\partial u}{\partial t} \right) = \nabla^2 \left(\frac{du}{dt} \right) - \nabla^2 \left(\vec{V} \cdot \nabla u \right)$$

So we can multiply by (-1) and substitute for $-\nabla^2 \left(\frac{\partial u}{\partial t} \right)$ in J:

$$J = -\nabla^2 \left(\frac{du}{dt} \right) + \nabla^2 \left(\vec{V} \cdot \nabla u \right) - \vec{V} \cdot \nabla \left(\nabla^2 u \right)$$

Now since the jet streak is a local maximum of u, we have ① $\nabla u \approx 0$ so the 2nd term ≈ 0.

② If we ignore the propagation of the jet streak, i.e., assume that the jet streak is not moving, then $\vec{V} \cdot \nabla \left(\nabla^2 u \right) \approx 0$ and the 3rd rhs term ≈ 0:

$$J \approx -\nabla^2 \left(\frac{du}{dt} \right)$$

- Recall we can write the eqn of motion as

  dV/dt = -fkxVa

Or just for the u-component,

  du/dt = fva

Then, $J \approx -\nabla 2\ (du/dt) = -\nabla^2(fv_a)$

Expand $\nabla^2(fv_a) = \partial^2/\partial x^2(fv_a) + \partial^2/\partial y^2(fv_a) + \partial^2/\partial p^2(fv_a) =$

$\partial/\partial x[\partial/\partial x(fv_a)]+\partial/\partial y[\partial/\partial y(fv_a)]+\partial/\partial p[\partial/\partial p(fv_a)]$

$=\partial/\partial x[\cancel{\partial f/\partial x(v_a)}+f\partial v_a/\partial x]+\partial/\partial y[\partial f/\partial y(v_a)+f\partial v_a/\partial y] + f\partial^2 v_a/\partial p^2$

- Taking $\partial f/\partial y=\beta$, and assuming the β-plane approximation,

$=f\partial^2 v_a/\partial x^2 + \partial/\partial y[\beta v_a+f\partial v_a/\partial y] +f\partial^2 v_a/\partial p^2$

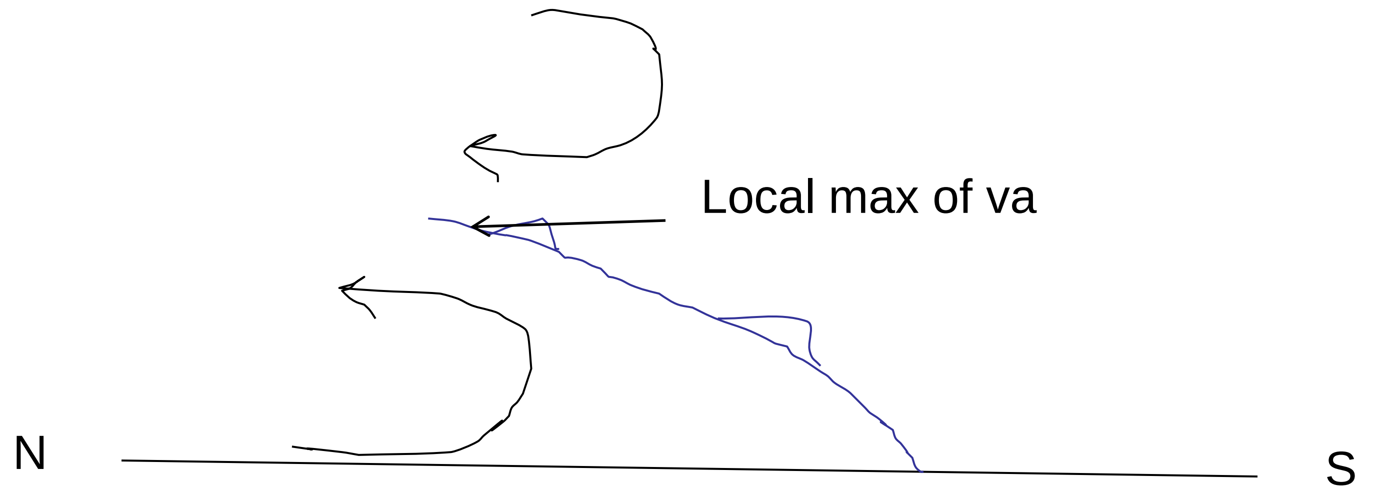
$= f\partial^2 v_a/\partial x^2 + \beta\partial v_a/\partial y + [\beta\partial v_a/\partial y + f\, \partial^2 v_a/\partial y^2\,]$
$+f\partial^2 v_a/\partial p^2$

$=f(\nabla^2 v_a)+2\beta\partial v_a/\partial y$

So that $J\approx -f\nabla^2 v_a - 2\beta\partial v_a/\partial y$

Notice that ∇^2 of something increases as the spatial scale becomes small; e.g., for $a=\sin(kz)$, we have $\nabla^2 a=-k^2\sin(kz)$ so that $\nabla^2 a$ increases as the wavenumber k increases (or as wavelength $Lz=2\pi/k$ decreases).

- Since the vertical dimension of the jet streak is small, we expect $\nabla^2 v_a$ to be large. For jet streaks of typical dimensions, and for typical wind speeds, the 1st RHS term will generally dominate.
- Therefore, we expect the jet streak to occur at a local maximum in the transverse (cross-jet) component of the ageostrophic wind. This tends to happen when an upper-level front approaches a sfc front: the ageostrophic thermally-direct circulation of the sfc front phases with the ageostrophic thermally-indirect circulation that tends to occur around upper-level fronts



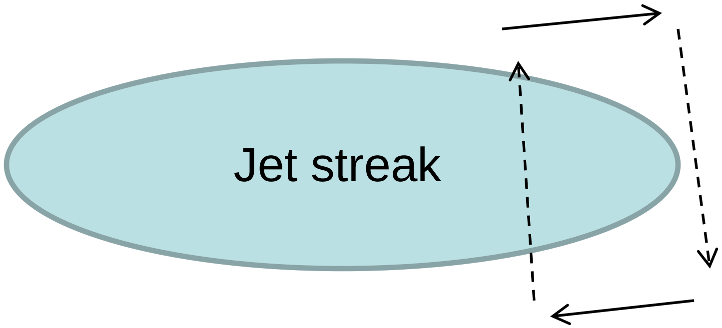
- We can also make some other qualitative statements about jet streaks

*A jet streak may be produced when a midlatitude trough encounters a subtropical ridge – this causes the height contours to pack together where the bottom of the trough and the top of the ridge meet

Jet influences on severe weather

- Consider low-level convergence/divergence due to jet streaks (part of va and vg are non-divergent)
- $V_a = 1/f\ k \times \{\partial V_g/\partial t + \partial V_a/\partial t + (V_g+V_a)\nabla(V_g+V_a) + \omega\partial V_g/\partial p + \omega\partial V_a/\partial p\}$

The part due to $\partial V_g/\partial t$ is the isallobaric/isallohypsic wind



Cold advection aloft due to Va<0
Warm advection below due to Va>0
Moist advection below/dry advection aloft
Rising air cools →increases low-level potential instability

Also can think of the rising air in left-exit region causing pressure falls, inducing isallobaric Va – creating low-level jet at a big angle from the upper jet, which is favorable for severe weather

Sawyer- Eliassen eq.

Take

The thermodynamics equation is

$$\frac{D_p\theta}{Dt} + \omega\frac{\partial\theta}{\partial p} = 0 \;, \tag{9.42}$$

where

$$\frac{D_p}{Dt} = \frac{\partial}{\partial t} + (\boldsymbol{v}_g + \boldsymbol{v}_a)\cdot\nabla_p \;.$$

~~That is, in contrast to the quasi-geostrophic thermodynamics equation (9.21), advection of temperature by the ageostrophic part of the wind is retained.~~

earlier eqn

$V_a = \frac{1}{f} \times \frac{DV_g}{Dt}$

* For simplicity it will be assumed that the front is oriented along the x axis. Furthermore, suppose the front is straight, i.e., there are accelerations only along it, so that $Dv_g/Dt \approx 0$ and hence from (9.41) $u_a \approx 0$ (Shapiro, 1981); assume that there is no curvature vorticity so that $\partial v_g/\partial x = 0$, and $f = f_o$.

$\frac{d\vec{v}_g}{dt} = -f\,(\hat{k}\times\vec{V}_a) \Rightarrow \vec{v}_a =$

also, neglect variations in f. After combining the x-components of (9.41) and (9.42) and using the equation of continuity and assuming that the thermal wind relation holds, it is found that (Sawyer, 1956; Eliassen, 1962; Shapiro, 1981)

$\frac{du_g}{dt} = -fv_a$

$$\frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{\kappa}\frac{\partial}{\partial y}\left(-v_a\frac{\partial\theta}{\partial y} - \omega\frac{\partial\theta}{\partial p}\right) + \frac{\partial}{\partial p}\left[-v_a\left(f_0 - \frac{\partial u_g}{\partial y}\right) + \omega\frac{\partial u_g}{\partial p}\right]$$
$$= 2\left(\frac{\partial u_g}{\partial p}\frac{\partial v_g}{\partial y} - \frac{\partial v_g}{\partial p}\frac{\partial u_g}{\partial y}\right) - \frac{R}{C_p f_0 p}\frac{\partial}{\partial y}\left(\frac{dQ}{Dt}\right) \;. \tag{9.43}$$

This diagnostic equation relates the two dependent variables v_a and ω to the geostrophic wind field, the horizontal temperature field, diabatic heating, static stability, and absolute vorticity. The ageostrophic circulation lies in the y-p plane only. Finding solutions to (9.43) is simplified if a vertical stream function ψ is defined such that

$$v_a = -\frac{\partial\psi}{\partial p} \tag{9.44a}$$

and

$$\omega = \frac{\partial\psi}{\partial y}. \tag{9.44b}$$

Ψ = streamfn. for the ageostrophic part of the circ.
2 vars → single.

f_{max}

y

$\frac{\partial\omega}{\partial y}$ $\frac{\partial v}{\partial\varphi}$ − +

notice $\zeta \approx \frac{\partial\omega}{\partial y} + \frac{\partial v_a}{\partial p} = -\frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial p^2}$ $-\nabla^2\psi$

$\Rightarrow$

$\ell \uparrow$ $\Downarrow$

thermal wind rel: $-\frac{\partial \vec{v}_g}{\partial p} = \frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{R/c_p}(k\times\nabla_p\theta)$

Then, from (9.44) and (9.16), (9.43) becomes

$$\frac{\partial^2\psi}{\partial y^2}\left[-\frac{\partial\theta}{\partial p}\frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{\kappa}\right] + \frac{\partial^2\psi}{\partial y\,\partial p}\left(2\frac{\partial u_g}{\partial p}\right) + \frac{\partial^2\psi}{\partial p^2}\left(f_0 - \frac{\partial u_g}{\partial y}\right)$$

$$= 2\frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{\kappa}\left(\frac{\partial\theta}{\partial y}\frac{\partial v_g}{\partial y} + \frac{\partial\theta}{\partial x}\frac{\partial u_g}{\partial y}\right) - \frac{R}{C_p f_0 p}\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right). \qquad (9.45)$$

* Sawyer-Eliasse

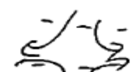
This linear, second-order equation, known as the Sawyer-Eliassen equation, relates the dependent variable ψ to the other independent variables; it is elliptic and therefore always has unique solutions if

$$\left(\frac{\partial u_g}{\partial p}\right)^2 + \left[\frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{\kappa}\right]\frac{\partial\theta}{\partial p}\left(f_0 - \frac{\partial u_g}{\partial y}\right) < 0\,. \qquad (9.46)$$

The right-hand side of (9.45) represents forcing:

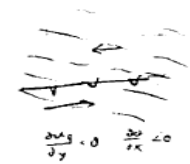
- Forcing due to changes in the across-the-front temperature gradient caused by geostrophic stretching deformation along the front (Fig. 9.13):

$$2\frac{R}{f_0 p}\left(\frac{p}{p_0}\right)^{\kappa}\left(\frac{\partial v_g}{\partial y}\frac{\partial\theta}{\partial y}\right).$$



- Forcing due to changes in the across-the-front temperature gradient as geostrophic shearing deformation tilts the along-the-front temperature gradient into the cross-front direction (Fig. 9.14): (twisting)

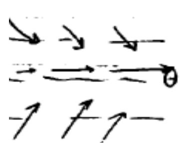
$\frac{\partial u_g}{\partial y}\frac{\partial\theta}{\partial x}$

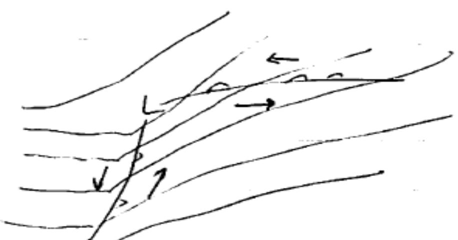


- Forcing due to differential diabatic heating:

$$-\frac{R}{C_p f_0 p}\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right).$$

Some differences in the structure of warm fronts and cold fronts have been explained in terms of the second kind of forcing (Eliassen, 1962; Gidel, 1978). Along warm fronts its effect is often frontolytical, and along cold fronts its effect is often frontogenetical (Fig. 9.15).





Friction and latent heat release can have some influence on fronts.

~~Upper level fronts depend more on~~ $\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial p}$