Low Level Jet
Occurrence

- Most common in late spring and early summer
- Direction is usually from S or SW
- Nocturnal maximum in speeds at least twice that of afternoon minimum (peaks usually reach 30-50 knts at around 925 mb height)
- Important ingredient for MCCs, 75% of all squall lines
Causes

• Inertial Oscillation due to frictional decoupling in early evening
• Baroclinicity produced in inclined boundary layer

\[ \text{warm} \quad \text{---} \quad \text{cool} \]

• Air currents blocked by Rockies
• Large-scale baroclinicity
• Interactions between upper-level jet streaks and diabatic processes associated with cyclogenesis
Notes

- Isallobaric component due to lee cyclogenesis
- Heating causes pressure gradient to change
- Soil moisture has an impact along with moisture gradients along sloping terrain (dry ground leads to increased speeds but less upward motion at end due to less clouds)
Simple solution for low-level jets

• Begin with horizontal equations of motion:
  \[
  \frac{Du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv
  \]
  \[
  \frac{Dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu
  \]

Assume the p.g.f. is constant in time

Break the wind into geostrophic and ageostrophic components: \( u = u_g + u_a, \quad v = v_g + v_a \)
• Then we can rewrite the momentum equations as

\[
\frac{d}{dt}(u_g + u_a) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + f v_g + f v_a
\]

\[
\frac{d}{dt}(v_g + v_a) = -\frac{1}{\rho} \frac{\partial p}{\partial y} - f u_g - f u_a
\]

• Recall the geostrophic wind components are given by:

\[
u_g = -\frac{1}{\rho f} \frac{\partial p}{\partial y} \text{ since } \frac{1}{\rho f} \frac{\partial p}{\partial y} = -f u_g
\]

\[
v_g = \frac{1}{\rho f} \frac{\partial p}{\partial x} \text{ since } \frac{1}{\rho f} \frac{\partial p}{\partial x} = f v_g
\]

And since we assumed the pgf is constant, the geostrophic wind must be constant as well \((du_g/dt = dv_g/dt = 0)\)
• Then we can substitute for the pgf in the momentum equations to get:

\[ \frac{Du_a}{dt} = -fv_g + fv_g + fv_a = fv_a \]

\[ \frac{Dv_a}{dt} = fu_g - fu_g - fu_a = -fu_a \]

• We can reduce this system to one eqn in one unknown by defining the complex wind vector \( \mathbf{V} = u + iv \). Then we have:

\[ \frac{d\mathbf{V}}{dt} = \frac{d}{dt} (u + iv) = \frac{du}{dt} + i \frac{dv}{dt} \]

• So we get \( \frac{dV}{dt} \) by multiplying the 2\(^{nd}\) eqn by \( i \) an adding to the first
• In terms of this complex vector, ignoring a,
\[ \frac{du}{dt} = fv \]
\[ +idv/dt = -ifu \]
\[ (du/dt + idv/dt) = -f(iu-v) \]
\[ = -if(u+iv) \text{ (note } (-if)(iv) = fv) \]

Or \[ \frac{DV}{Dt} = -ifV \]

Integrate \[ d\frac{V}{V} = d(lnV) = -ifdt \]
\[ \ln(V(t)/V_0) = -if(t-t_0) \]

Assume \( t_0=0 \) is our initial time; then
exponentiate to get \[ V(t) = V_0e^{-ift} \]
What does this solution look like?

• Recall the Euler relations:
  
  \[ e^{i\alpha} = \cos(\alpha) + i\sin(\alpha) \]
  
  \[ e^{-i\alpha} = \cos(\alpha) - i\sin(\alpha) \]

  Here we have \(\alpha = ft\). Then we could rewrite the solution as:

  \[ \mathbf{V}(t) = \mathbf{V}_0[\cos(ft) - i\sin(ft)] \]

  So \(u = u_0 \cos(ft)\) and \(v = v_0 \sin(ft)\).
We can see that...

- The amplitude is conserved
- The solution oscillates such that the original phase is recovered at \( ft = 2\pi, 4\pi, \text{etc} \)

Then the period \( \tau \) of the oscillation is given by \( ft = 2\pi \) or \( \tau = 2\pi / f \). This is referred to as the inertial period. We see that \( \tau \) varies with latitude since \( f \) varies with latitude. Recall \( f = 2\Omega \sin \Phi \); while \( \Omega = 2\pi / \text{day} \). Then we have \( \tau = 2\pi / f = 2\pi (\text{day}) / 4\pi \sin \Phi = 1 \text{ day} / 2 \sin \Phi \). So the inertial period is 1 day when \( \sin \Phi = 1/2 \); ie \( \Phi = 30^\circ \).
• Numerous other processes have been cited to explain the LLJ. Research has shown that the LLJ is associated with the development of surface cyclones. This in turn can be caused by:

* lee side troughing
* adjustment to mass redistribution by upper level jet streaks
Application of equations

• If we know $\tau$ and $V_a$, we can figure out how the LLJ will behave overnight.
• Assume at sunset, $V$ is 10 ms$^{-1}$ from 135° and $V_g$ is 20 ms$^{-1}$ from 180°.
• First, need to determine $V_a$. We need to do this using vector subtraction (break $V$ and $V_g$ into components)

$$u_g = 0, \quad v_g = 20, \quad u = -7.1, \quad v = 7.1$$
• Thus \( u_a = u - u_g = -7.1 - 0 = -7.1 \text{ m/s} \)
• \( V_a = v - v_g = 7.1 - 20 = -12.9 \text{ m/s} \)
• So \( V_a \) is \((-7.1^2 + -12.9^2)^{0.5} = 14.5 \text{ m/s}\)
and direction is \( \tan^{-1}(u_a/v_a) = 29^\circ \).

\[ \tau = \frac{1 \text{ day}}{2 \sin \Phi} = 18.7 \text{ h}, \] so \( V_a \) will swing around and make a circle in that time.

• When will the peak wind be? It will be when \( V_a \) is in the same direction as \( V_g \). \( V_g \) was from 180, so it has to rotate 151 degrees. This will happen in \((151/360)(18.7 \text{ h})\), or about 8-9 hours after sunset.

• What will the peak speed be? Add \( V_g \) and \( V_a \). It would be \( 20 \text{ m/s} + 14.5 \text{ m/s} = 34.5 \text{ m/s} \).
Picture what is happening with the ageostrophic wind (and thus LLJ)

Note the LLJ veers during the night and since it is the source of heat/moisture for thunderstorm systems, they also veer in their path overnight too.