Chapter 4 – Radiative Transfer

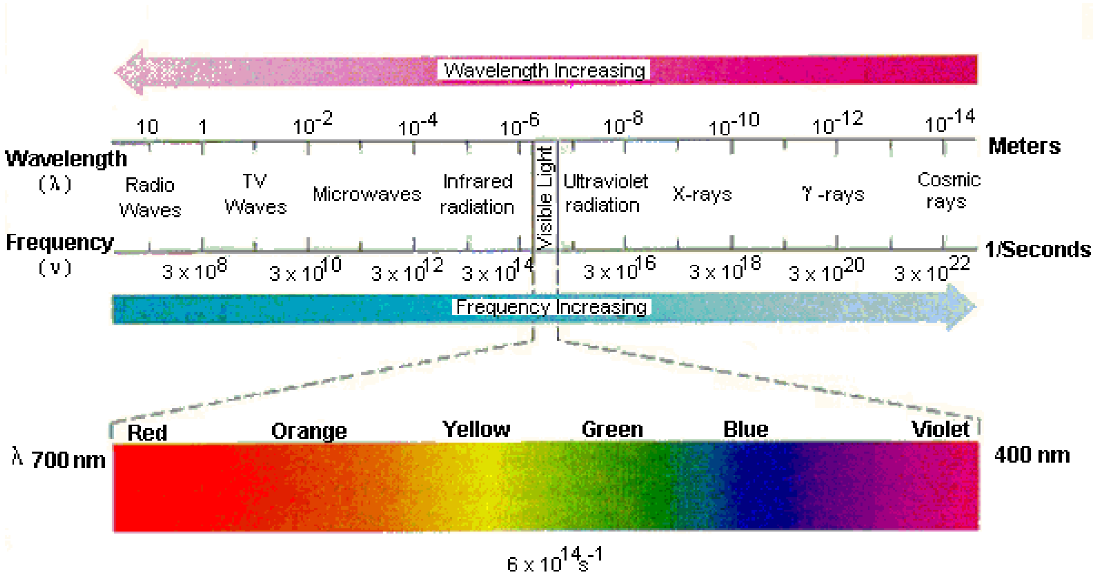
Spring 2024

Radiative Transfer

- Primary method of energy exchange between the Earth and rest of universe.
- Transfers occur between the atmosphere and Earth, and between layers of the atmosphere.

The Spectrum of Radiation

- Electromagnetic Radiation
  - Travels at speed of light ($3x10^8$ m/s).
  - Consists of a variety of frequencies and wavelengths.



Quantum Theory

- Electromagnetic radiation is made up of photons, or packets of energy.

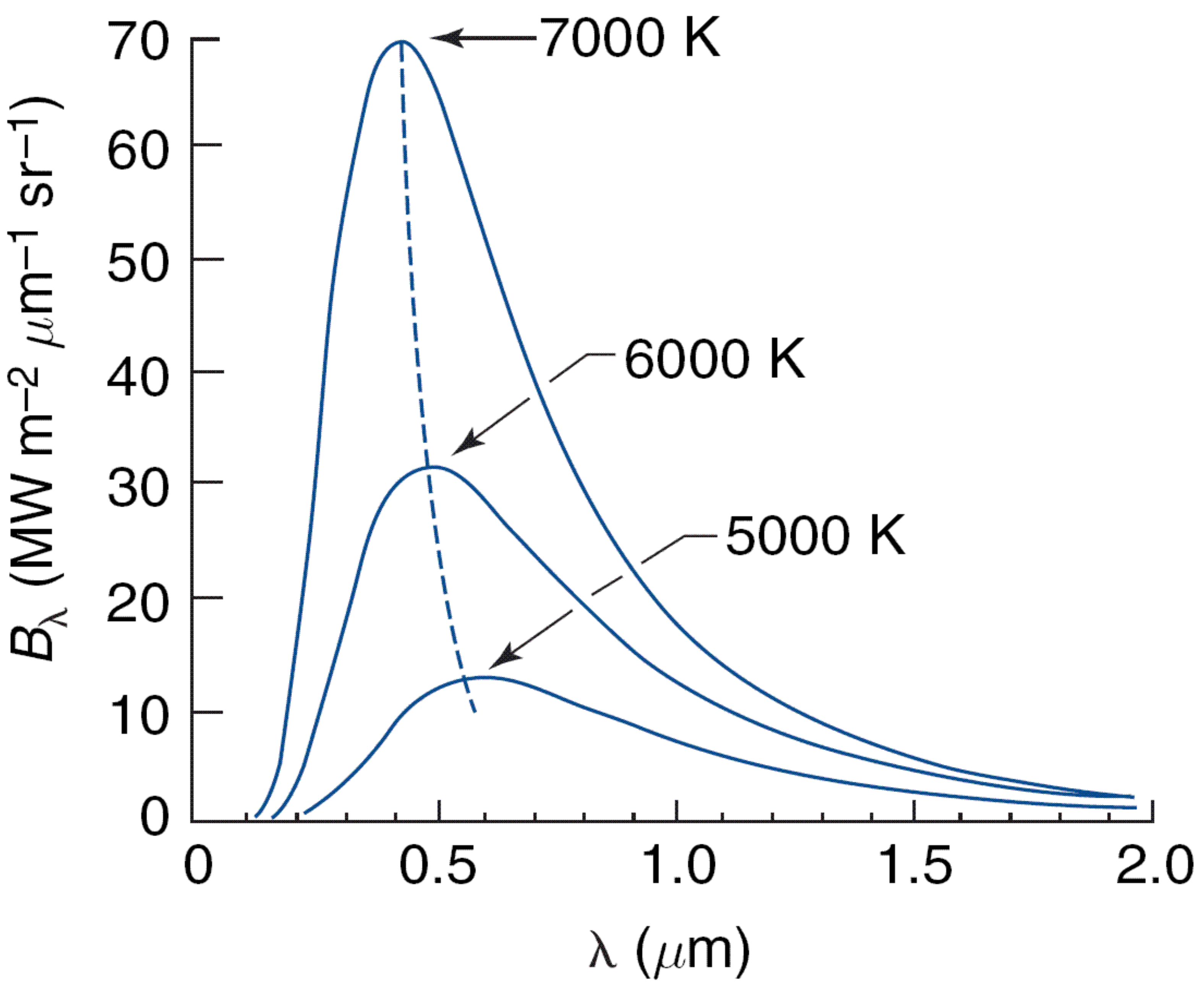
- Photon energy = W = hf = hc/λ
  - λ = wavelength
  - f = frequency
  - c = speed of light.

- Energy is inversely proportional to wavelength.

- Energy is directly proportional to frequency.

Nomenclature

- Radiant flux: Rate of energy transfer by electromagnetic radiation.
  - Units: Energy/time = J/s = W = Watts.
  - Example: Radiant flux from sun = 3.9×10^{26} W.
- Irradiance: Radiant flux/Area = E
  - Units: W/m^2
  - Example: Irradiance at outermost disk of sun.
- Monochromatic irradiance: E = E / λ
  - Units: W/m^2 m = W/m^2 um.



7000 K
6000 K
5000 K
70
60
50
40
30
20
10
0
0
0.5
1.0
1.5
2.0
B_λ (MW m^{-2} μm^{-1} sr^{-1})
λ (μm)

Diffuse and Direct Radiation

- Diffuse Radiation: Radiation emanating from a source that subtends a finite arc of solid angle.
  - Scattered radiation is an example.
- Parallel Beam Radiation: Emission from a concentrated source.
  - Radiance approaches infinity and the angle subtended by the source approaches zero.
  - Direct beam radiation

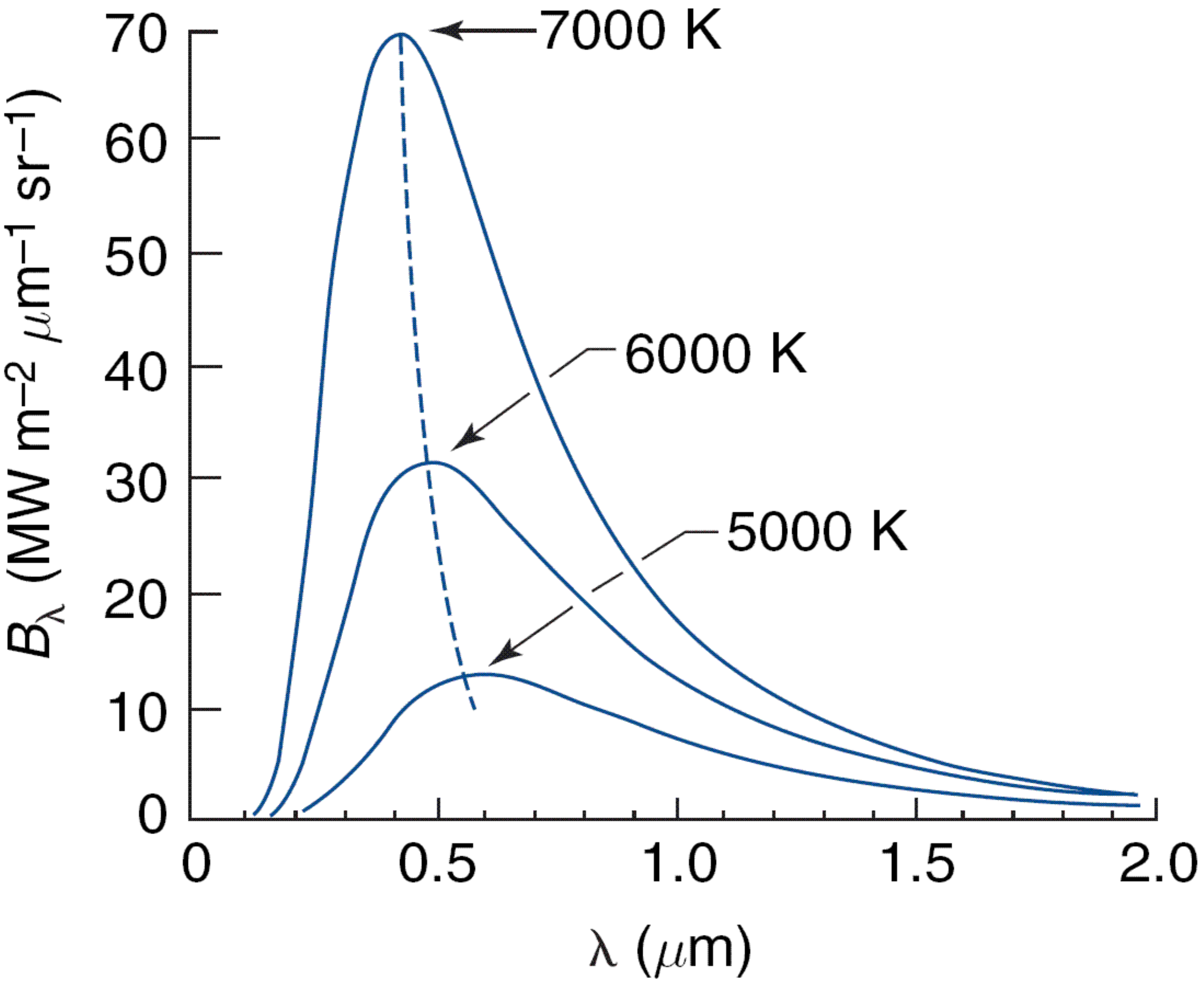
Measurement of Radiation

- Black and white surface
- Black absorbs radiation, white reflects radiation
- Amount of radiation received and absorbed determines the differences in the rate of increase of temperatures between the two surfaces.
- Pyranometer

Blackbody Radiation

- Hypothetical body comprising a sufficient number of molecules absorbing and emitting electromagnetic radiation in all parts of the spectrum so that:
  - All incident radiation is completely absorbed.
  - Maximum possible emission is realized in all wavelength bands, in all directions (isotropic).

- Planck's law: Amount of radiation emitted by a blackbody.
  - Uniquely determined by its temperature.



7000 K
6000 K
5000 K
B_λ (MW m^−2 μm^−1 sr^−1)
λ (μm)
0
10
20
30
40
50
60
70
0
0.5
1.0
1.5
2.0

Wien's Displacement Law

- λ_{max} = 2880 um K/T
- Wavelength of peak emission for a blackbody at temperature T.
- Estimate the temperature of a radiation source from its emission spectrum.
    - If we assume a blackbody source, then knowing the emission spectrum, we can deduce T.

More Blackbody Spectra

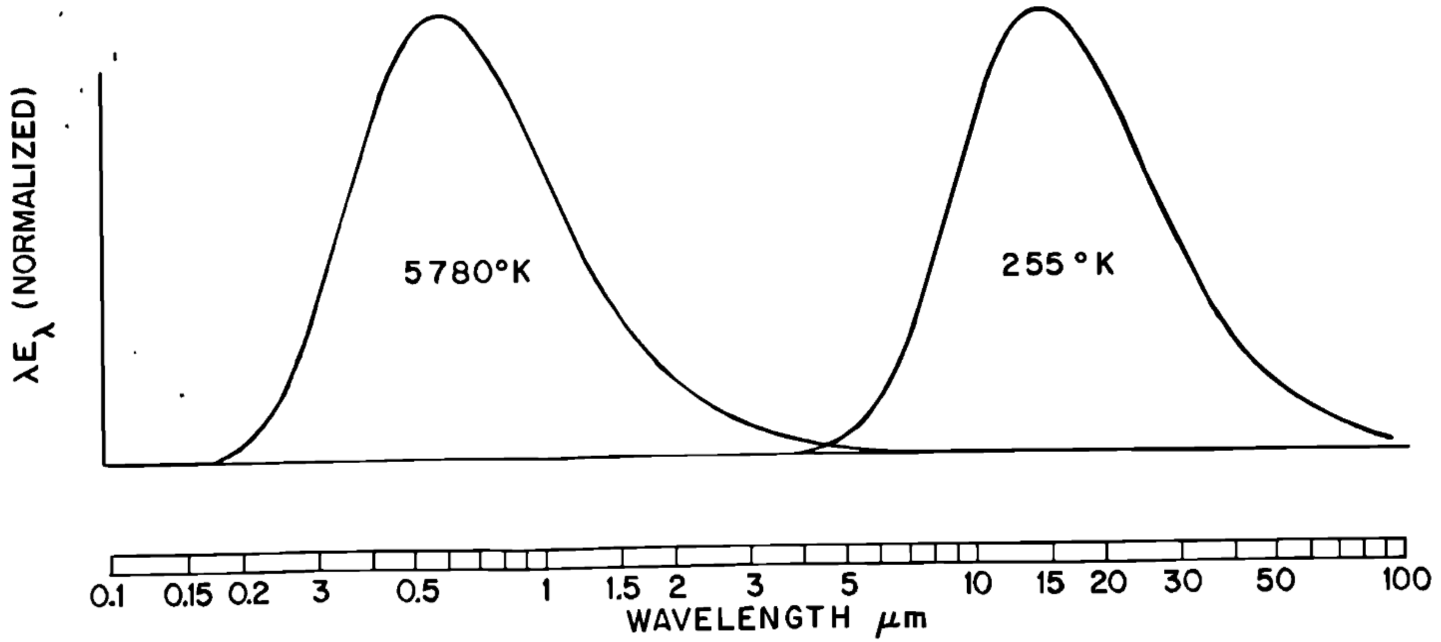


Fig. 6.4 Normalized blackbody spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength in order to make area under the curves proportional to irradiance. [Adapted from R. M. Goody, "Atmospheric Radiation," Oxford Univ. Press (1964), p. 4.]

Blackbody Spectrum cont.

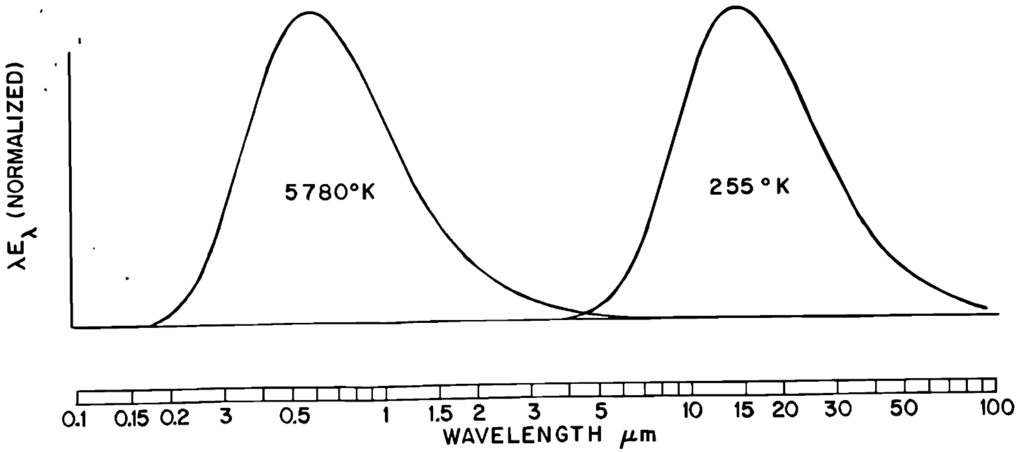
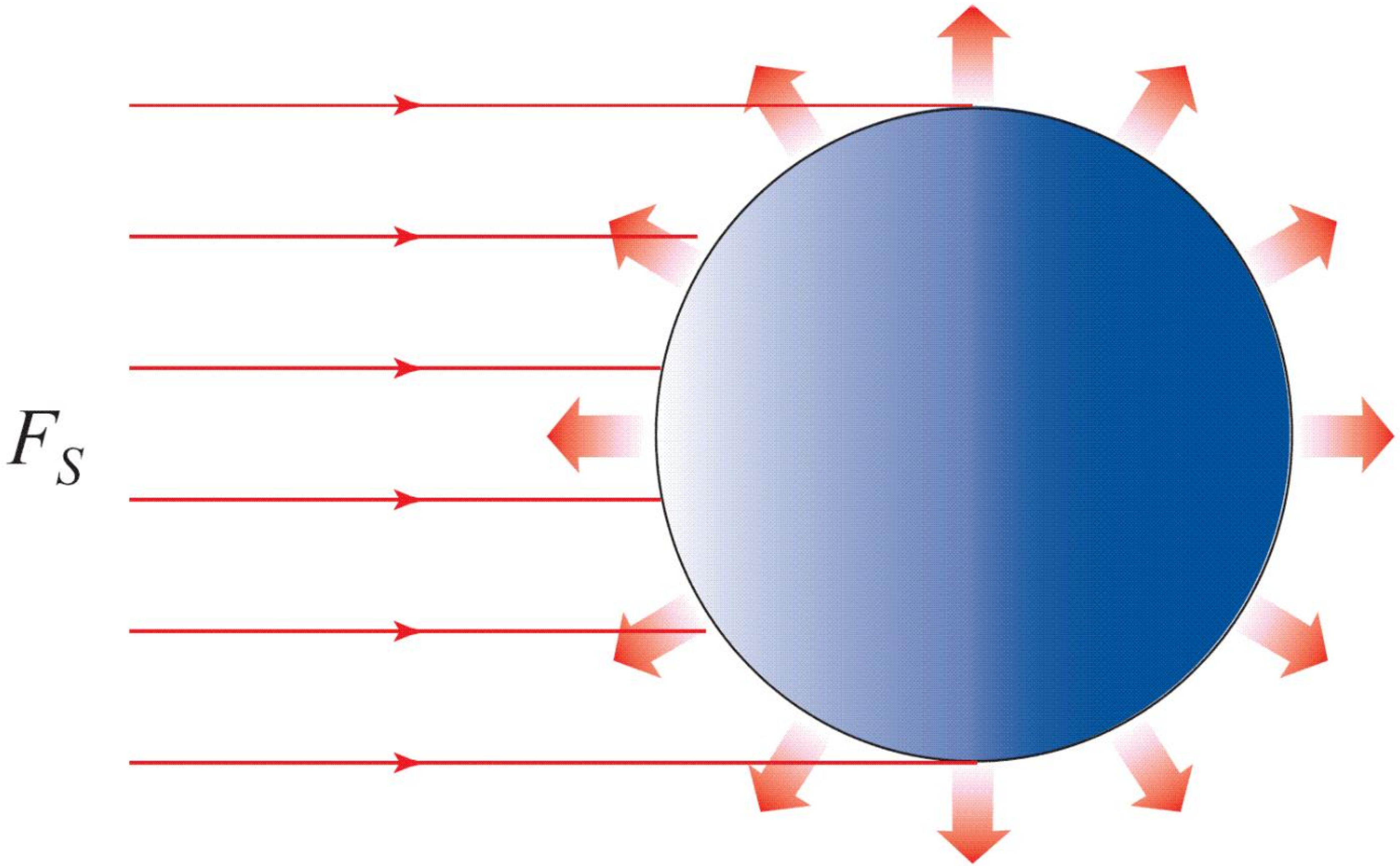


Fig. 6.4 Normalized blackbody spectra representative of the sun (left) and earth (right), plotted on a logarithmic wavelength scale. The ordinate is multiplied by wavelength in order to make area under the curves proportional to irradiance. [Adapted from R. M. Goody, "Atmospheric Radiation," Oxford Univ. Press (1964), p. 4.]

- Peak for sun is in blue, but asymmetry of spectrum gives more radiation toward yellow side.
- Earth emits @ ~255 K
- Sun concentrated in visible and near infrared, planets and their atmospheres largely confined to infrared.
- Note: Curves barely overlap
  - Treat solar (shortwave) radiation separately from terrestrial (longwave) radiation.

Calculations

- Irradiance at the top of the Earth’s atmosphere
- Equivalent Blackbody Temperature of Sun
- Equivalent Blackbody Temperature of Earth
  - This calculation assumes the Earth does not have an atmosphere and references the image on the next slide



F_S
F_E

Absorptivity and Emissivity

- Blackbody radiation is an upper limit to the amount of radiation a real substance may emit at a given temperature.
  - Real world radiation < Blackbody

- At any given wavelength, λ, we can define the Emissivity, $\varepsilon \equiv E_{\lambda} / E_{\lambda}^{*}$
  - Emissivity is a measure of how strongly a body radiates at that wavelength.
  - $\varepsilon_{blackbody} \equiv 1$ at all wavelengths.
  - $0 < \varepsilon_{real\ substance} < 1$

- "Grey body" emissivity: $\varepsilon \equiv E / E^{*} = E/\sigma T^4$ and $E_{grey} = \varepsilon \sigma T^4$
  - "Grey" comes from the neglect λ of wavelength dependence of the emissivity.
  - Most real substances behave as grey bodies and have an emissivity that is different from 1.

- Absorptivity, $a_{\lambda} \equiv$ irradiance absorbed / irradiance incident
  - "grey body" absorptivity = a
  - $a_{blackbody} = 1$

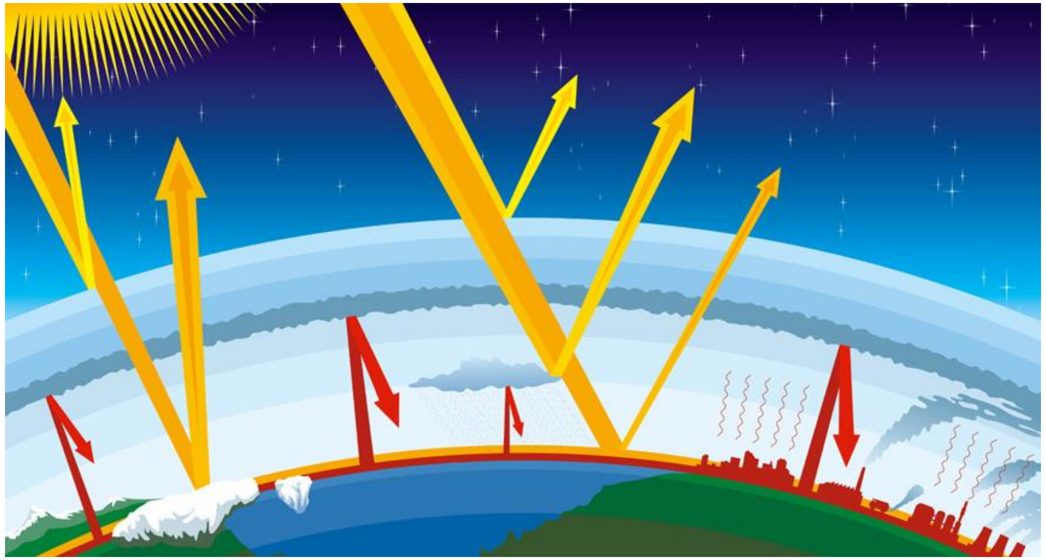
Kirchhoff's law

- Kirchhoff's law: Materials that are strong absorbers at a particular λ are also strong emitters at that λ.
- $a = \varepsilon$
- $a_\lambda = \varepsilon_\lambda$
- Weak absorbers = weak emitters
- Applies to gases like our atmosphere.

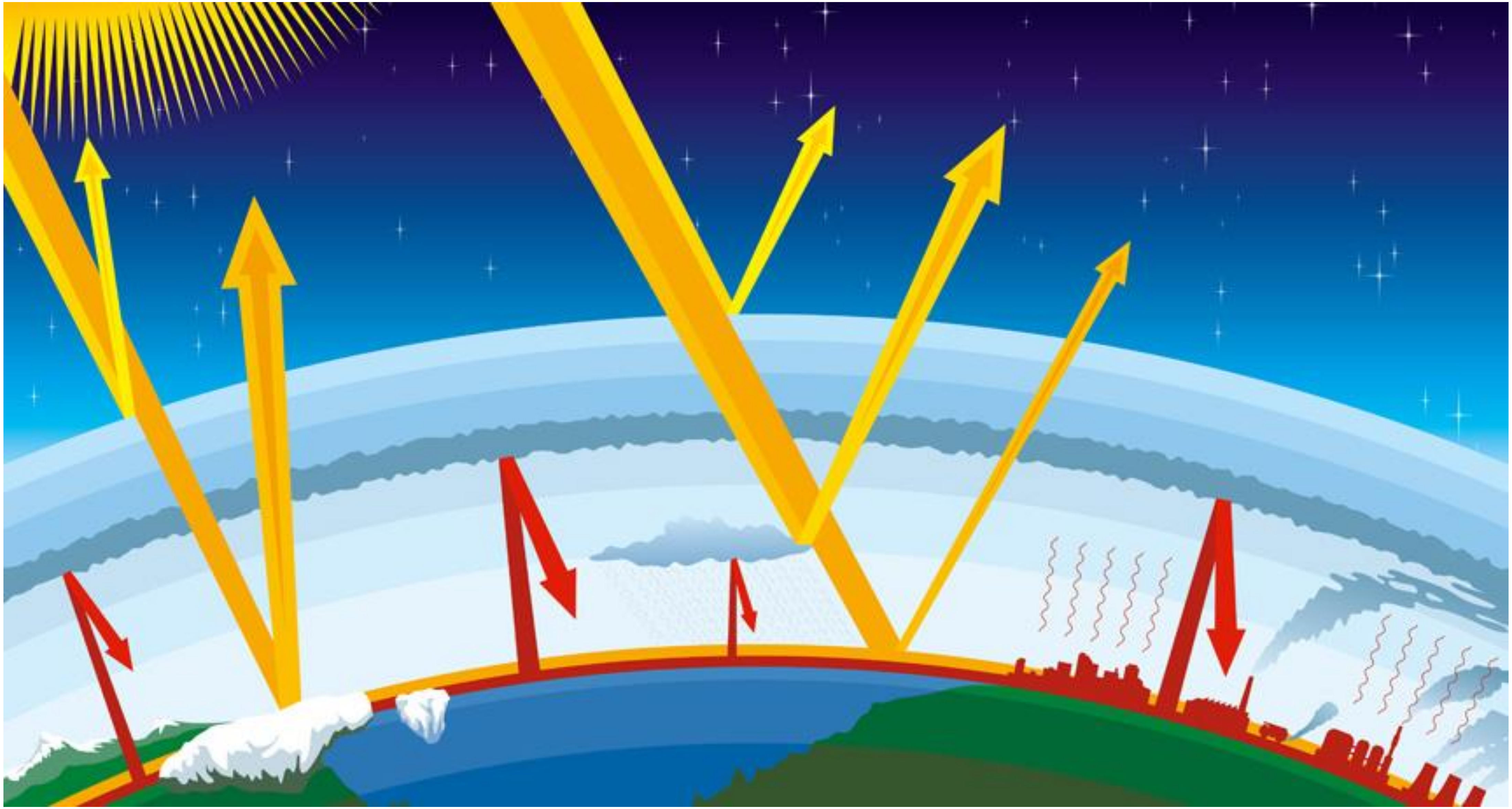
Reflectivity and transmissivity

- What happens to the part not absorbed? It is reflected.
- E_λ (incident) = E_λ (absorbed) + E_λ (reflected)
- Dividing by E_λ (incident) yields:
- E_λ (incident)/ E_λ (incident) = E_λ (absorbed)/ E_λ (incident) + E_λ (reflected)/ E_λ (incident)
- 1 = E_λ (absorbed)/ E_λ (incident) + E_λ (reflected)/ E_λ (incident)
- $1 = a_\lambda + r_\lambda$
- Reflectivity, r_λ = E_λ (reflected)/ E_λ (incident)
  - Large r_λ = small a_λ and vice versa.
- More generally, for non-opaque media, some of the incident radiation is transmitted.
- Transmissivity, τ_λ = E_λ (transmitted)/ E_λ (incident)
- $a_\lambda + r_\lambda + \tau_\lambda = 1$

Greenhouse Effect



- Solar radiation essentially passes through to surface.
- Atmosphere absorbs some of IR emitted by the surface and emits it back.
- Surface must warm up even faster to emit enough radiation so that output can match the input
  - Radiative equilibrium

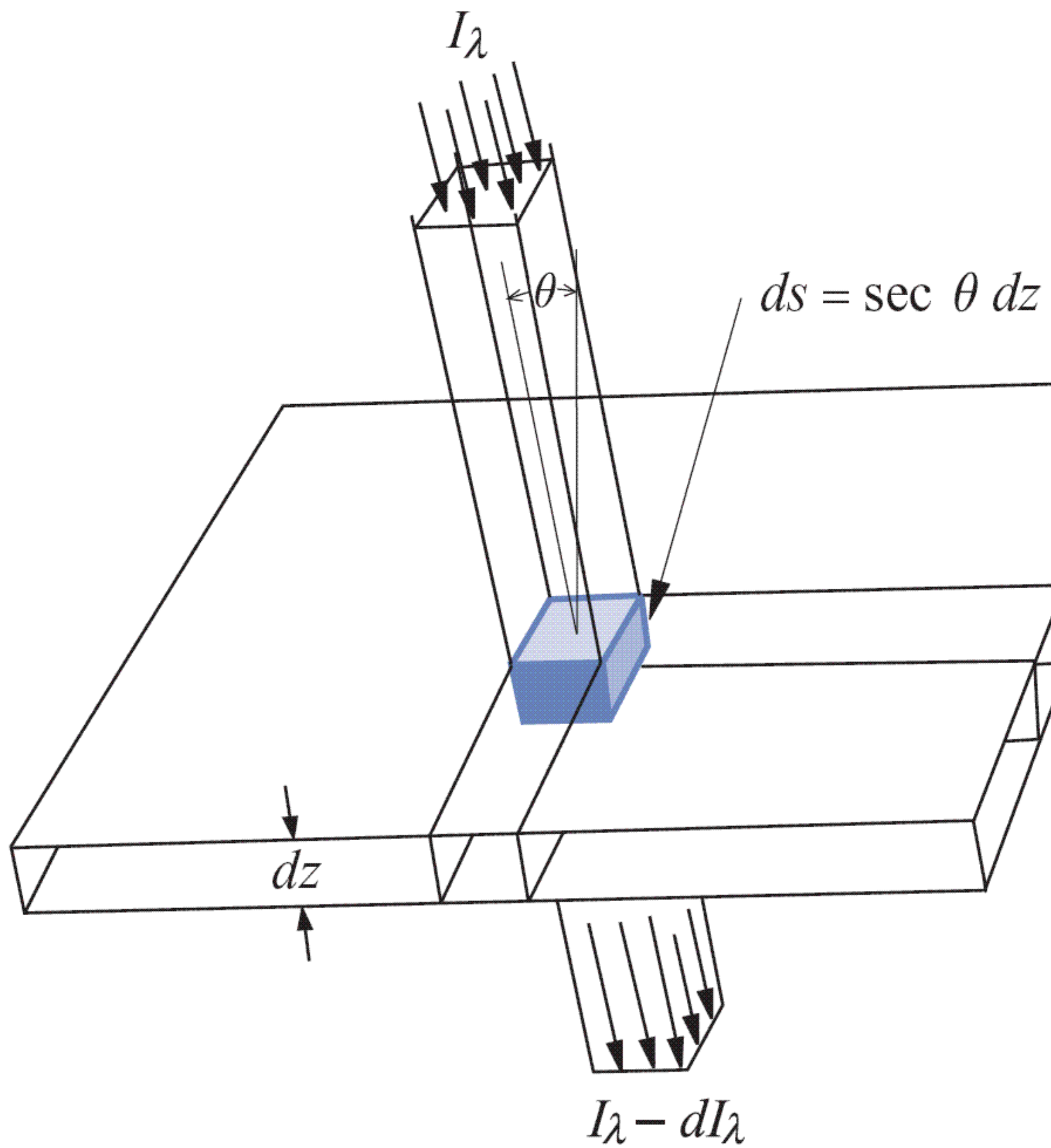


Atmospheric Absorption of Solar Radiation

- Absorption of parallel beam radiation is proportional to the number of molecules of gas along the path.
    - For now, we are going to ignore scattering of photons out of the beam.

- This can be expressed as: $da_\lambda = -dE_\lambda / E_\lambda = - K_\lambda \rho \sec \phi \, dz$
    - where:
    - da_λ is the absorption that occurs through the layer.
    - ρ is the density of the gas
    - $\sec \phi \, dz$ is the path length (see online lecture)
        - $\rho \sec \phi \, dz$ is the mass per unit area for a small dz (think about units)
    - K_λ is the absorption coefficient of the gas [m^2/kg]
        - How efficient the gas is as an absorber
        - Also called the absorption cross-section
        - K_λ is a function of the temperature of the gas, pressure, and composition of the gas.

- There are three ways to change the amount of absorption:
    - Change the density of the gas (more absorbers per unit area)
    - Change the path length
    - Change the absorption coefficient

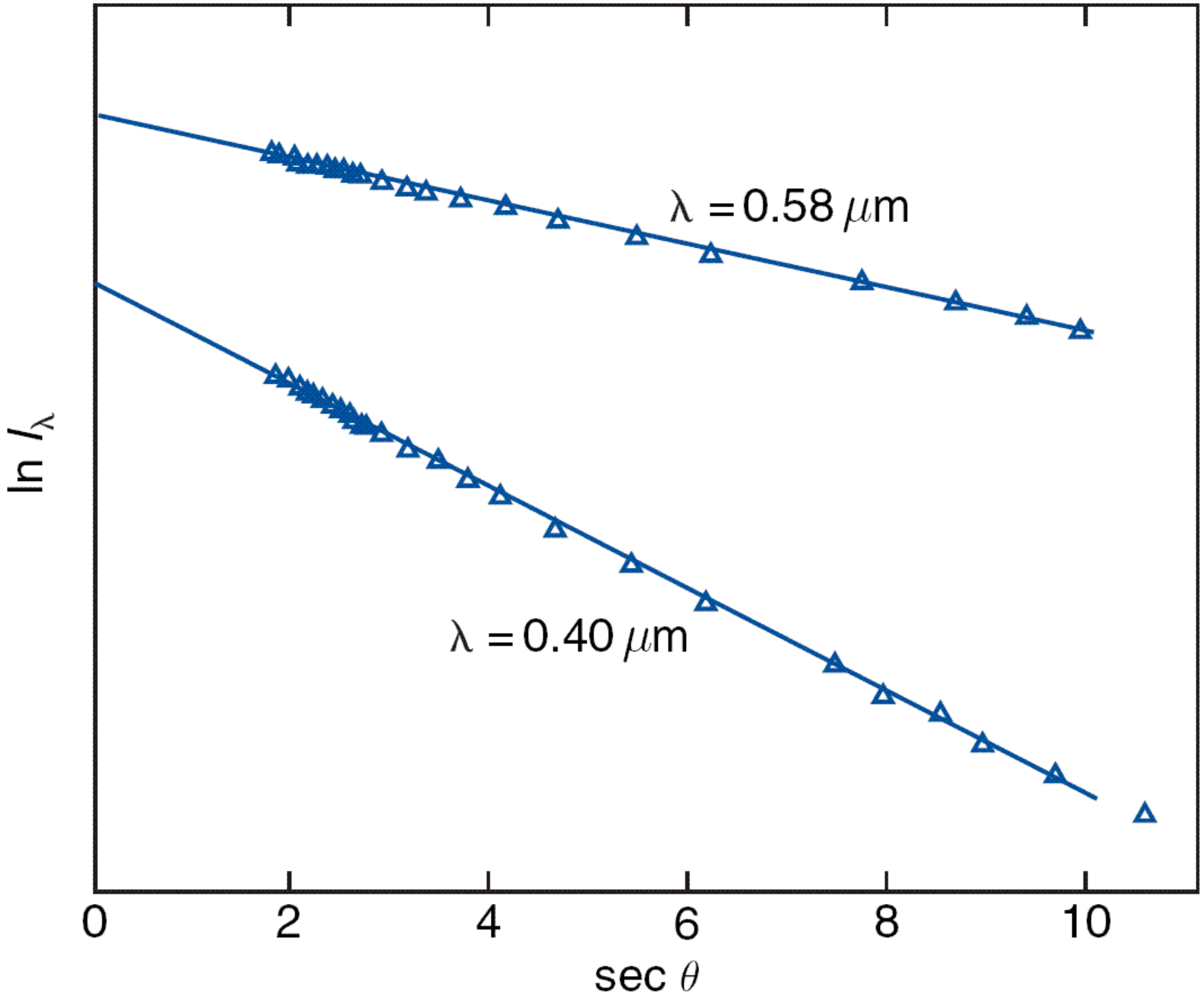


I_λ
θ
$ds = \sec\ \theta\ dz$
dz
$I_\lambda - dI_\lambda$

Beer’s Law and Transmissivity

- See in class lecture deriving Beer’s law and how it relates to the transmissivity

- Beer’s law is an equation for the cumulative absorption or how much of the radiation remains after passing through a given thickness of the atmosphere.



[From *J. Appl. Meteor.*, **12**, 376, (1973).]

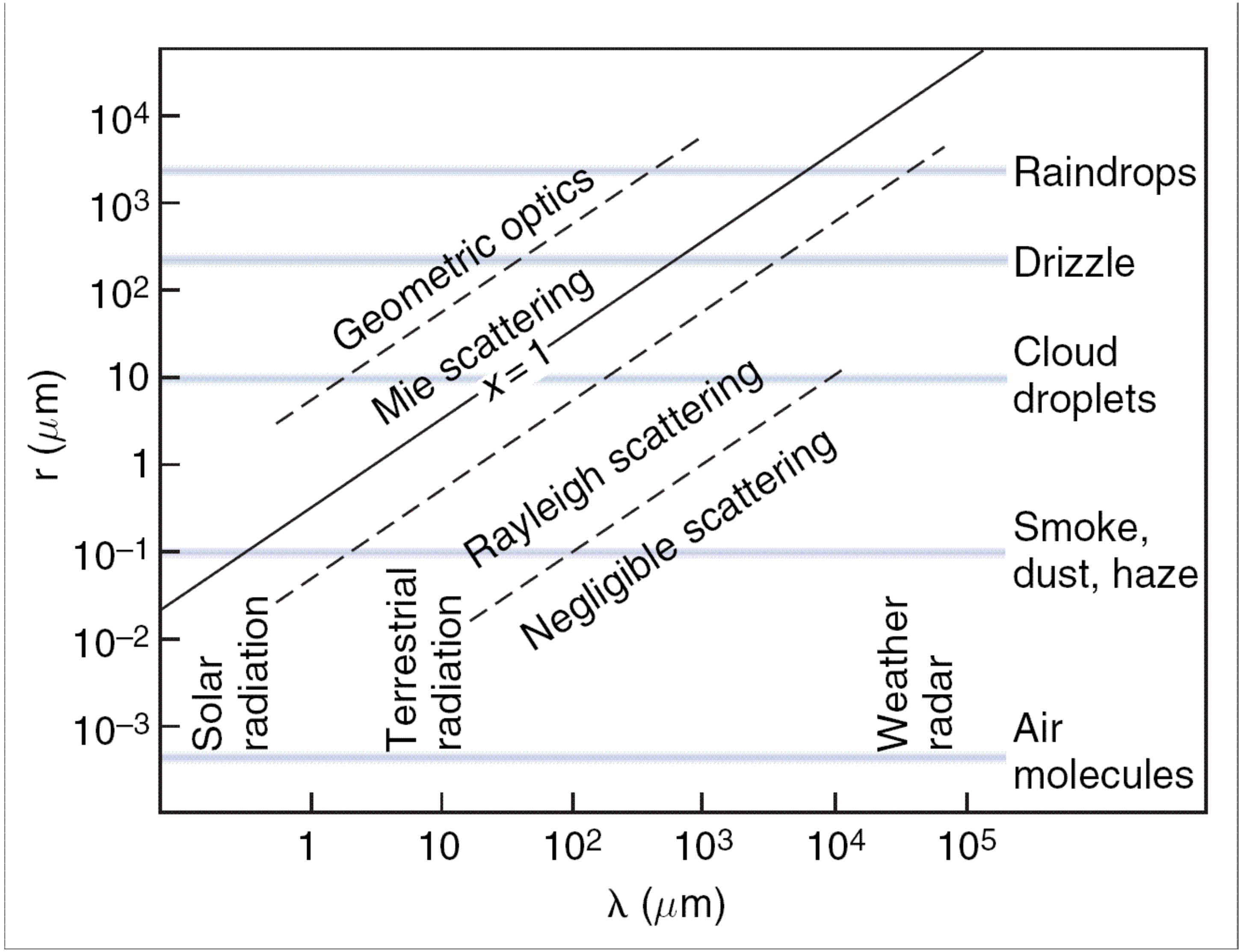
Atmospheric Scattering of Solar Radiation

- ds_λ = Fraction of parallel beam radiation that is scattered when passing downward through a layer of infinitesimal thickness.
- This can be expressed as: $ds_\lambda = -dE_\lambda / E_\lambda = - K_\lambda N \sigma \sec \phi \, dz$
  - where:
  - ds_λ is the scattering that occurs through the layer.
  - N is the number of particles per unit volume of air (particle density).
  - $\sec \phi \, dz$ is the path length (see online lecture).
  - σ scattering cross-sectional area of each particle.
  - K_λ is the scattering coefficient of the gas [m^2/kg]
    - How efficient the gas is at scattering
    - K_λ is a function of the size parameter and the refractive index of the particles in the gas.

- There are four ways to change the amount of scattering:
  - Change the number of particles per unit volume.
  - Change the path length.
  - Change the scattering coefficient.
  - Change the scattering cross-section of the particles.

- By following the approach we took with absorption, you can come up with an equation similar to Beer's law, but for scattering.

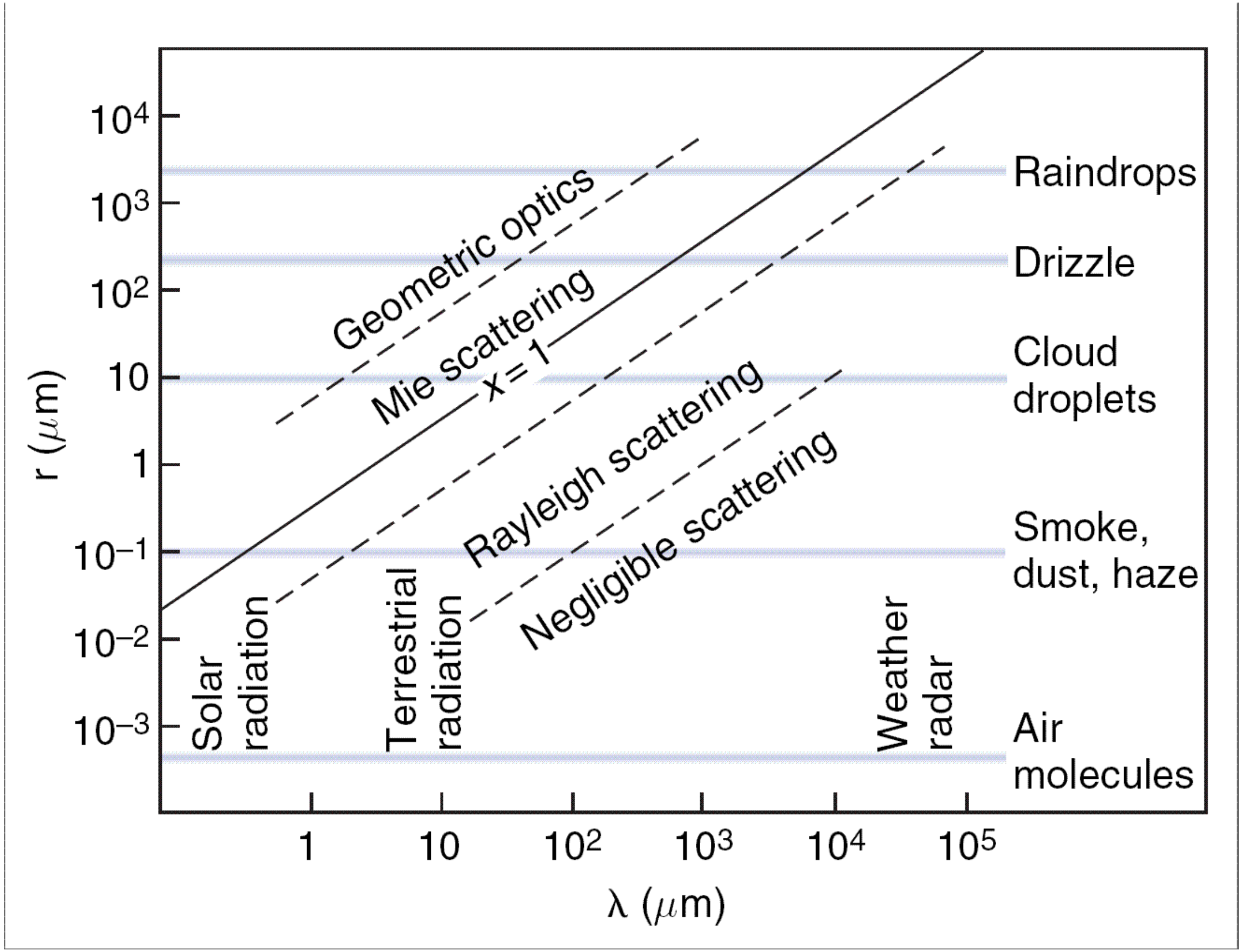
Scattering, x, and λ



$x \ll 1$, $K_\lambda \propto x^4$, and $K_\lambda \propto \lambda^{-4}$

- For size parameters much less than 1, $K_\lambda \propto x^4$ and $K_\lambda \propto \lambda^{-4}$
- Rayleigh scattering of solar (shortwave) radiation.
- Scattered radiation is evenly divided between forward and back-scattered hemispheres.
- K_λ(blue, λ= 0.47 um)/K_λ(red, λ =0.64 um) = $(0.64/0.47)^4$
- Short λ light is scattered more than long λ.
- Short λ is preferentially scattered.
  - Responsible for blue sky.
- Longer λ is more readily transmitted
  - Reddish or orange appearance of objects.
  - Especially around sunrise and sunset.
  - Path length through the atmosphere is long.

Scattering, x, and λ

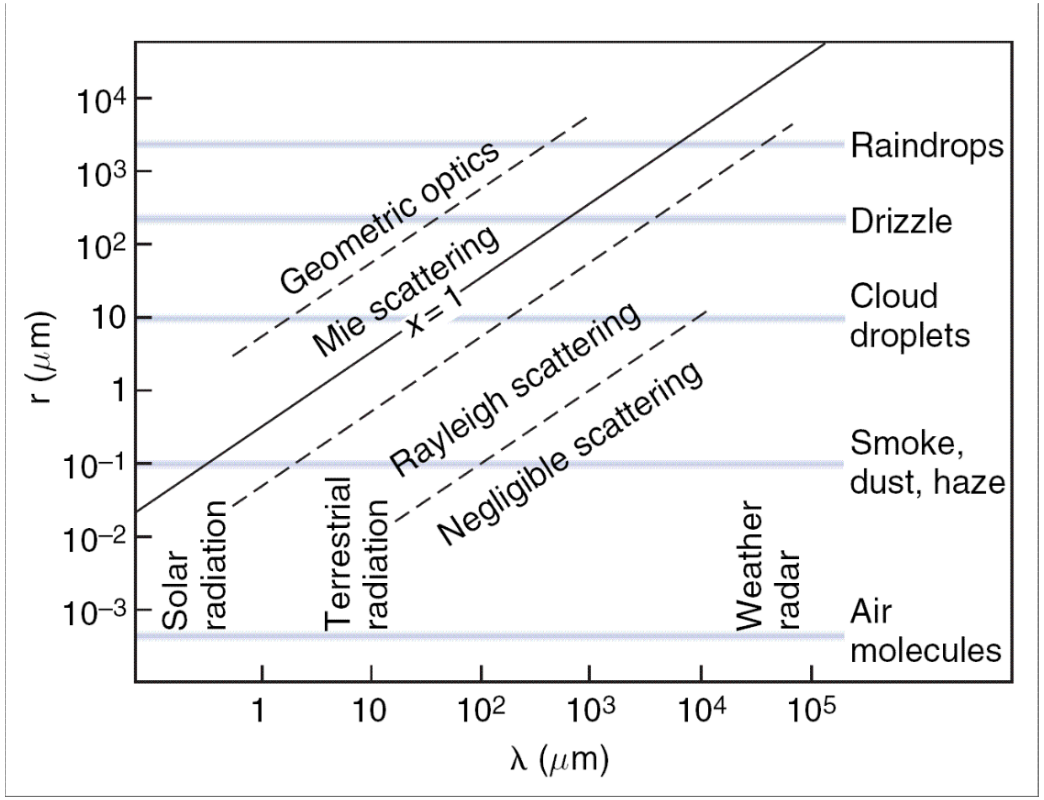


Microwave Scattering

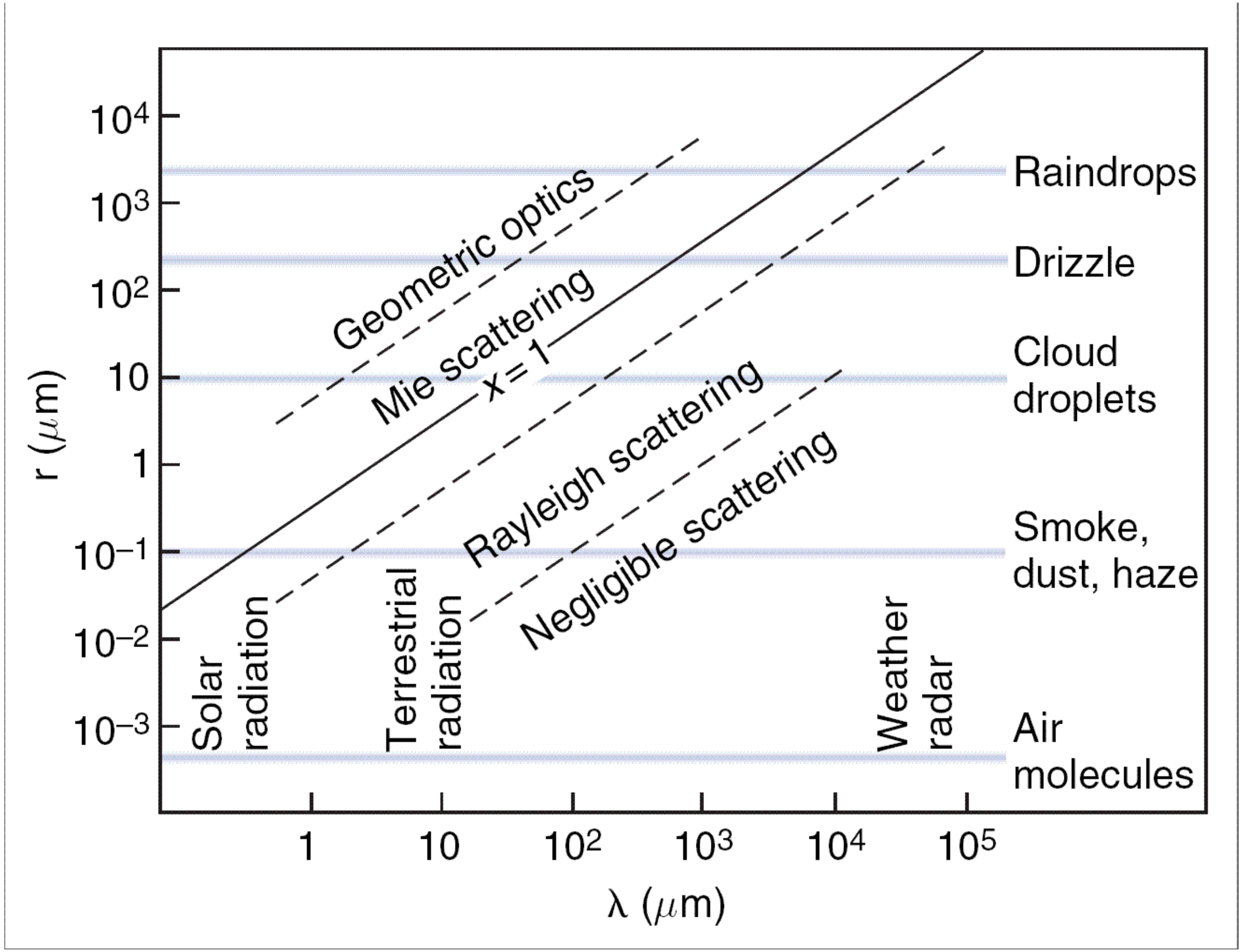
- Microwave scattering by raindrops also falls in the Rayleigh regime.
- For a given λ, $K_\lambda \propto x^4$.
- Sharp increase in K_λ with increasing drop size.
- Makes it possible to discriminate between precipitation and cloud drops (radar).
- Why not use infrared radiation?

Doppler Radar Bands

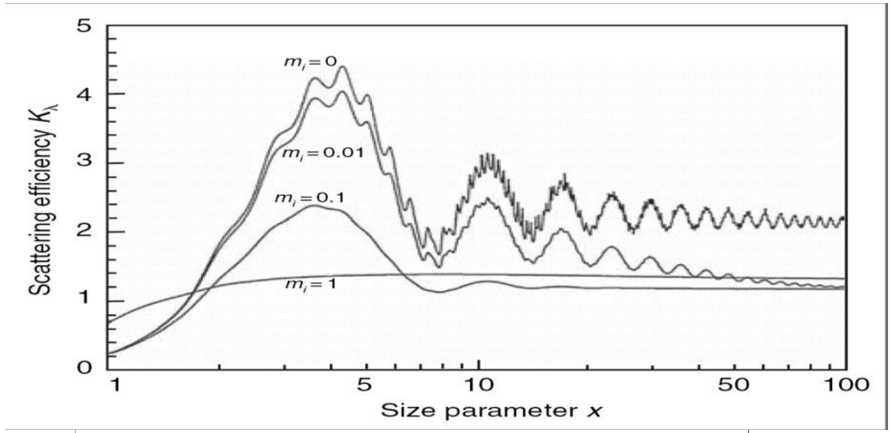
- S band
  - 8-15 cm wavelength
  - Not easily attenuated
  - NWS radars – 10 cm
- C band
  - 4-8 cm wavelength
  - More easily attenuated
  - Smaller dish sizes make them more affordable
  - TV stations
- X band
  - 2.5-4 cm wavelength
  - Easily attenuated, useful for short range observation
  - Cloud development
  - DOWs
- K band
  - 0.75-1.2 cm wavelength
  - Similar to X band, but even more sensitive
  - Shares space with police radars.

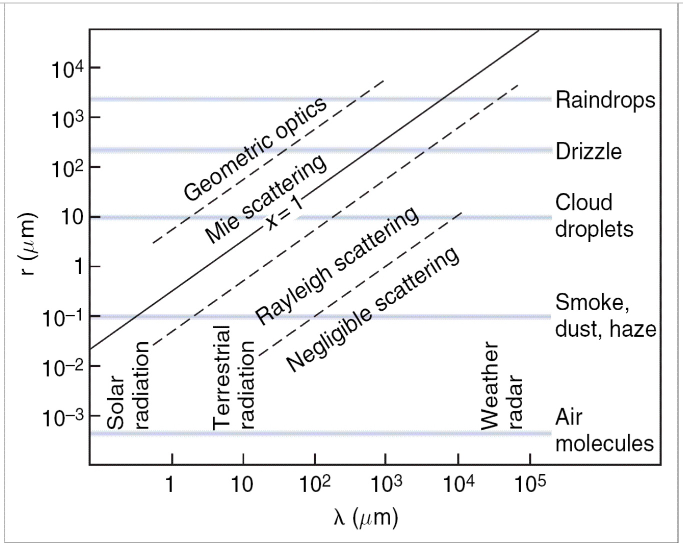


Scattering, x, and λ



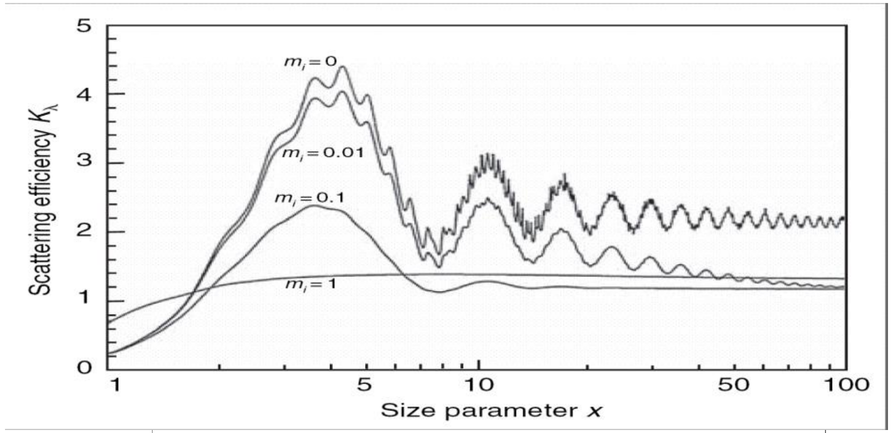
$x > 50$, $K_\lambda \cong 2$

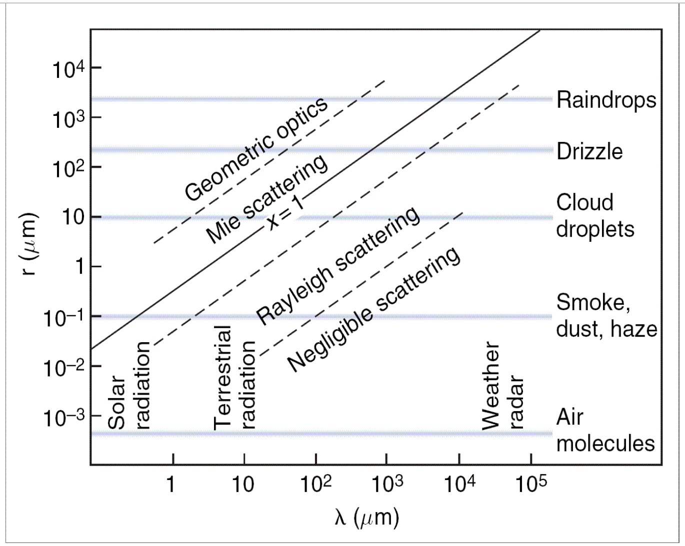




- Angular distribution of scattered radiation described by principles of geometric optics.
- Scattering of visible radiation by cloud droplets, rain drops, ice particles.
- Rainbows, halos, etc.

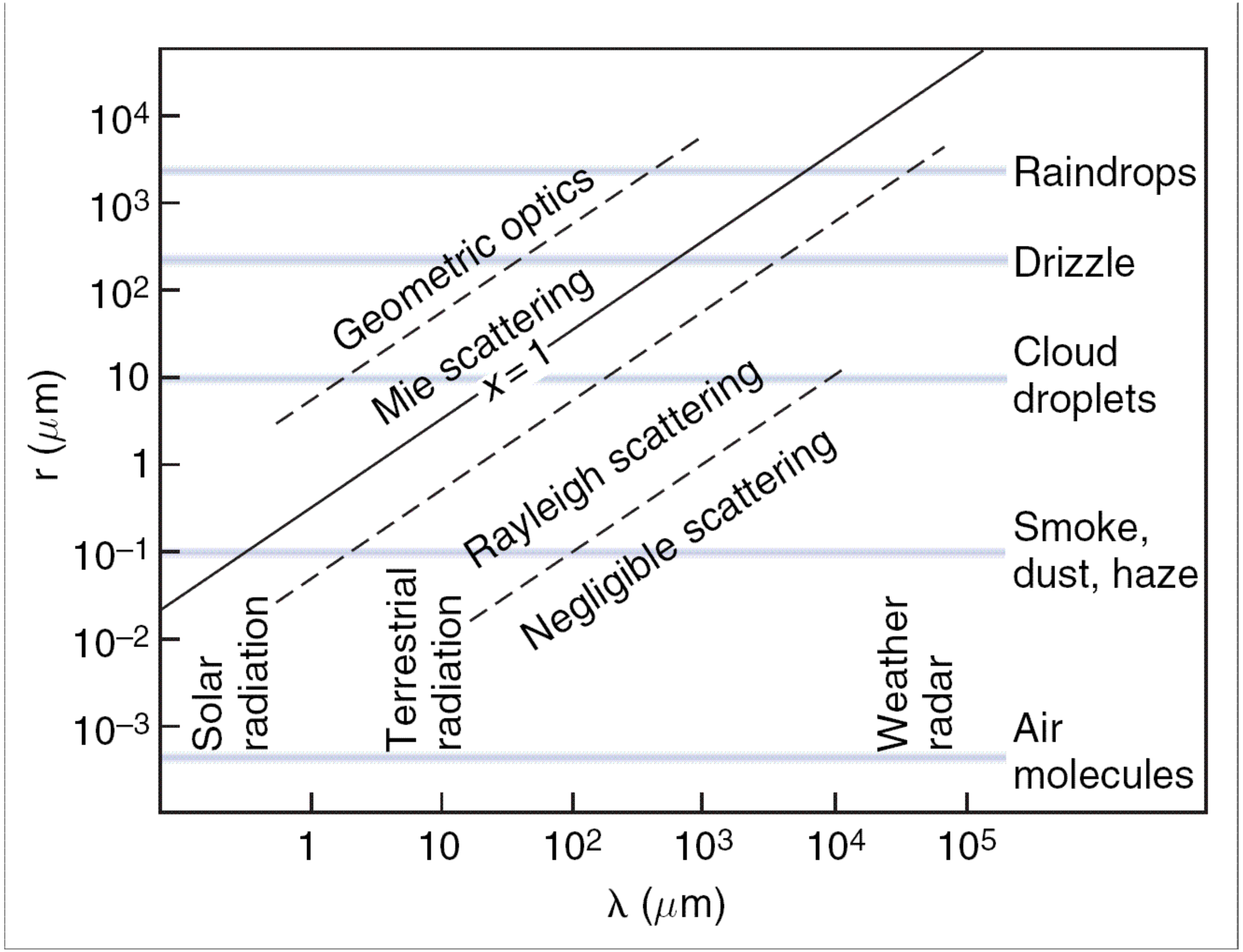
0.1 < x < 50





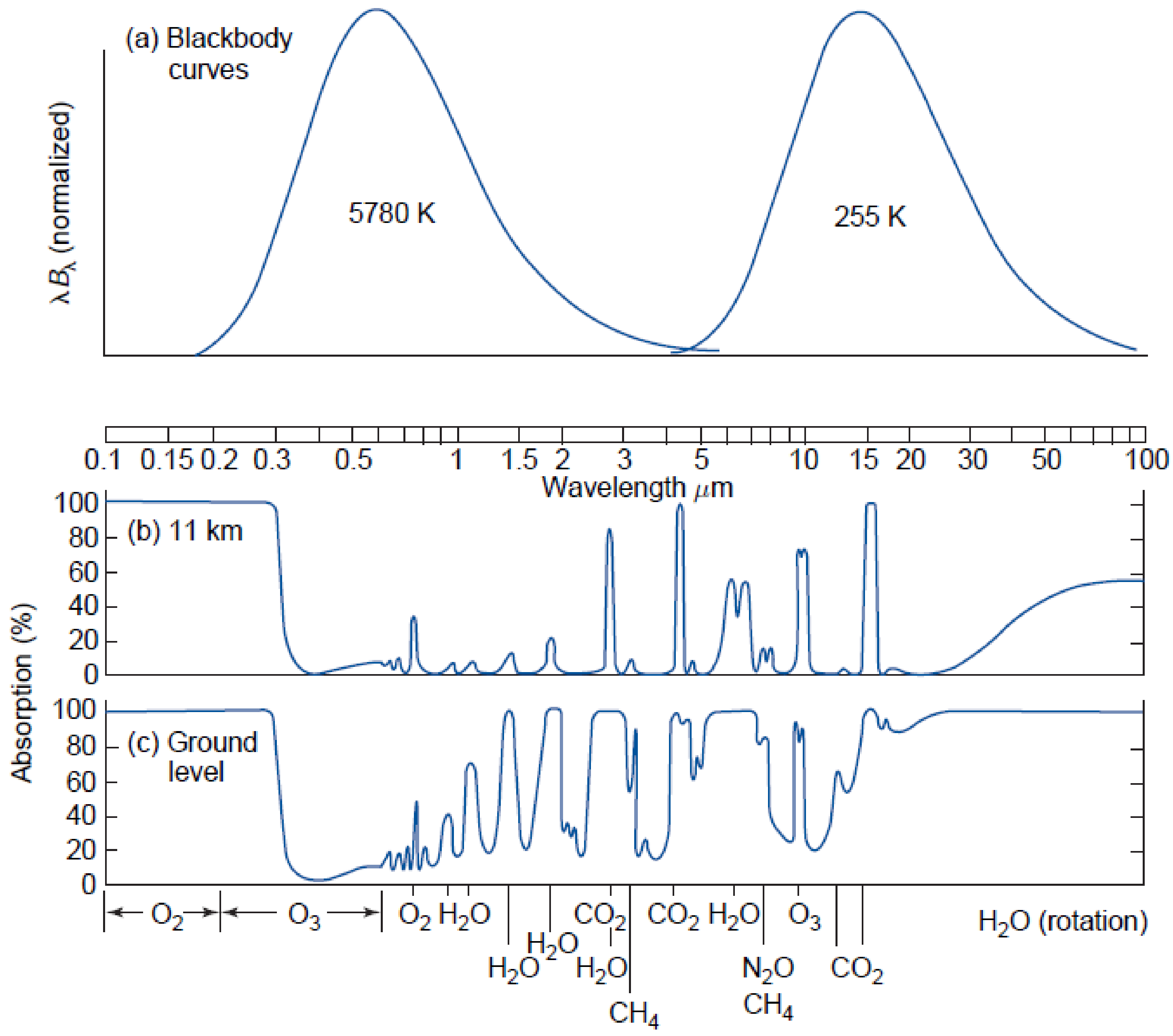
- K_λ exhibits oscillatory behavior
- Angular distribution of radiation very complicated and varies rapidly with the size parameter.
- Forward scattering predominates over back scattering.
- Scattering of sunlight by smoke, smog, dust.

Scattering, x, and λ



Extinction

- Equations for scattering and absorption are very similar.
- In fact, they can be made to be identical with the following equation:
- K_λ(Extinction) = K_λ(Scattering) + K_λ(Absorption)
- This equation gives the combined effect of scattering and absorption in depleting the intensity of radiation passing through the layer.



(a) Blackbody curves
λBλ (normalized)
5780 K
255 K
0.1 0.15 0.2 0.3 0.5 1 1.5 2 3 5 10 15 20 30 50 100
Wavelength μm
Absorption (%)
100 80 60 40 20 0
(b) 11 km
(c) Ground level
O2 O3 O2 H2O H2O H2O H2O CO2 CH4 CO2 H2O N2O CH4 O3 CO2
H2O (rotation)